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Anomalous Inferred Viscosity and Normal Density Near the ^{3}He T_c in a Torsion Pendulum

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Abstract
Precise measurements of the dissipation and resonant frequency of a torsion pendulum reveal an anomaly in the inferred viscosity and normal density of liquid ^{3}He near the superfluid transition. We present an argument that the anomaly originates in the large viscosity and large viscosity change of the normal component in the torsion tube in the vicinity of the superfluid transition.

Keywords Viscosity of ^{3}He · Normal fraction of ^{3}He · Torsion Pendulum

1 Introduction

Torsional oscillators were applied to the study of the viscosity and superfluid fraction of ^{3}He starting with the introduction of the high Q torsion pendulum by Reppy and co-workers [1, 2]. The anisotropy of the superfluid fraction [1], viscosity of ^{3}He near T_c [2], and behavior of the "bare" superfluid fraction over a wide range in temperature [3] were among the earliest studies enabled by the development of this instrumental technique. The observation of the Kosterlitz–Thouless transition in ^{4}He films [4], early observations on the superfluid behavior of disordered ^{4}He [5], ^{3}He-^{4}He mixtures [6], and ^{3}He [7], were also notable successes engendered by the high Q pendulum. It was also applied to the study of ^{3}He under strong regular confinement [8]. The technique represented a significant advance over the originating "Andronikashvili" torsional oscillator [9] and was characteristic of the sort of innovation that was inculcated by Reppy to his students. Those were not the only benefits we derived. He served as mentor, guide and inspiration for graduate students and postdocs. I recall experiencing great satisfaction as we (Parpia and Reppy) took data in the afternoon on critical velocities [10] after corralling every HP X-Y plotter in the laboratory. It required choreographing paper changing with the balancing of bridges and ramping of temperature. The joy of discovery that follows the

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expenditure of months of preparation can be immensely rewarding, and this is something we all strive to communicate to this day.

In the course of the measurements of viscosity and superfluid fraction, the torsion pendulum device was driven at a constant amplitude of motion (to avoid effects due to nonlinearities) at its resonant frequency using a phase locked loop. The resonant frequency (or its inverse, the period) and the drive amplitude needed to maintain a constant amplitude of motion were monitored. The drive amplitude is proportional to the dissipation (Q^{-1}).

This paper describes an anomalous result observed in the course of experiments performed several decades ago [2, 11], that has not been previously discussed. A qualitative explanation of the result is also presented.

2 Results

2.1 Normal Fluid Behavior

In the early application to the study of ^{3}He, the torsion tube (typically made of heat treated beryllium-copper alloy) was mated to an epoxy "head" in which a parallel plate cavity was formed. The height d of the parallel plate cavity was chosen so that the viscous penetration depth $\delta = \sqrt{(2\eta_n/\rho_n\omega)} \gg d$, where η_n is the viscosity, ρ_n is the density of the normal fluid, and ω is the angular frequency of the motion. For a fluid undergoing shear in contact with an oscillating plane, the viscous penetration depth characterizes the decay length for the velocity field in the fluid. The large viscosity of ^{3}He ($\sim$ 1 poise (0.1 Pa-s) near T_c) and low density ($\sim$ 0.1 g/cm^3 (100 kg m^{-3})) lead to a viscous penetration depth of order 500 μm at a kHz frequency near T_c. In this "well-locked" regime, a larger viscosity leads to increased overlap of the shear waves originating from each surface of the cavity in the torsional pendulum, and results in a higher Q. The fluid's inertia is also better coupled to the pendulum.

In Fig. 1, the resonant period of the ^{3}He filled torsion pendulum is plotted against the drive voltage. As the temperature decreases, the viscous penetration depth increases as T^{-1} due to the Fermi liquid's T^{-2} temperature-dependent viscosity increase. At some temperature, the two shear velocity profiles from the upper and lower surfaces of the torsion pendulum start to overlap sufficiently ($d/\delta \approx 2.25$) and the dissipation reaches a maximum. Any further increase in viscosity produces more locking of the fluid in the cavity and the velocity profile becomes parabolic and progressively "flatter" as δ increases in the well-locked fluid regime. Thus in the normal state, as the ^{3}He is cooled, the drive voltage increases at first with lower temperatures, attains a maximum and then decreases. All the while the period increases monotonically. This behavior is illustrated in Fig. 1 with high temperatures on the left and low temperatures on the right of the plot. All the behavior discussed near T_c is in the well-locked regime.

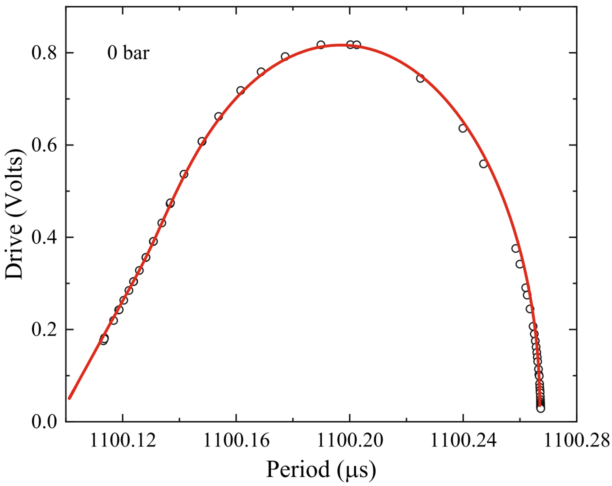

Fig. 1 Resonant period of the torsional pendulum plotted against the drive voltage required to maintain a constant amplitude of motion for normal ^{3}He contained between two parallel plates from ≈200 mK to 1.5 mK at 0 bar. At high temperature (smaller period), the fluid inertia is only weakly coupled to the pendulum. As the temperature decreases, the increased viscosity leads to more coupled fluid inertia and a monotonic increase in period (lower frequency). At high temperatures, the drive voltage increases as the viscosity increases (more dissipation). Eventually, the shear waves emanating from each surface begin to overlap, and the dissipation decreases. At low temperatures, the fluid enters into the well-locked regime where a larger viscosity leads to a lower dissipation. Here, the data were taken at 0 bar and confined in a 95 μm high cavity (Color figure online)

2.2 Superfluid Behavior

The experiments were carried out using the adiabatic demagnetization of cerium magnesium nitrate (CMN), a dilute electronic paramagnetic salt that orders at ≈ 1.5 mK. The CMN was powdered and packed into a beryllium-copper chamber and infused with ^{3}He liquid. A thermometer located in a tower weakly coupled to the demagnetization stage monitored the susceptibility of CMN using a SQUID sensor. The experiment was mounted above the thermometry tower. In most experimental runs, the magnetic field decreased over an hour or so to achieve the ultimate temperature and the experiment cooled into the superfluid phases. Data were collected while the cell warmed up under the ambient heat leak over the course of 3-30 h, depending on the pressure [2]. We note that $\eta_c, \rho_n = 1$ was designated in the original publication [2] as the value above which the viscosity and normal density attain nearly constant values, and we retain that designation here.

Part of the experimental program was aimed at exploring the temperature region near T_c. To acquire more data near T_c, a Helmholtz coil pair generating ∼ 2.5 mT was introduced and programmed by a separate controller after the main magnet achieved the lowest temperature. The current in these coils could be varied allowing the temperature to be ramped at rates around 1 nK/s. To achieve sufficient precision in the period measurements, the signal was averaged over 10^5 periods, (≈100 s); thus 20 points were obtained in about an hour, during

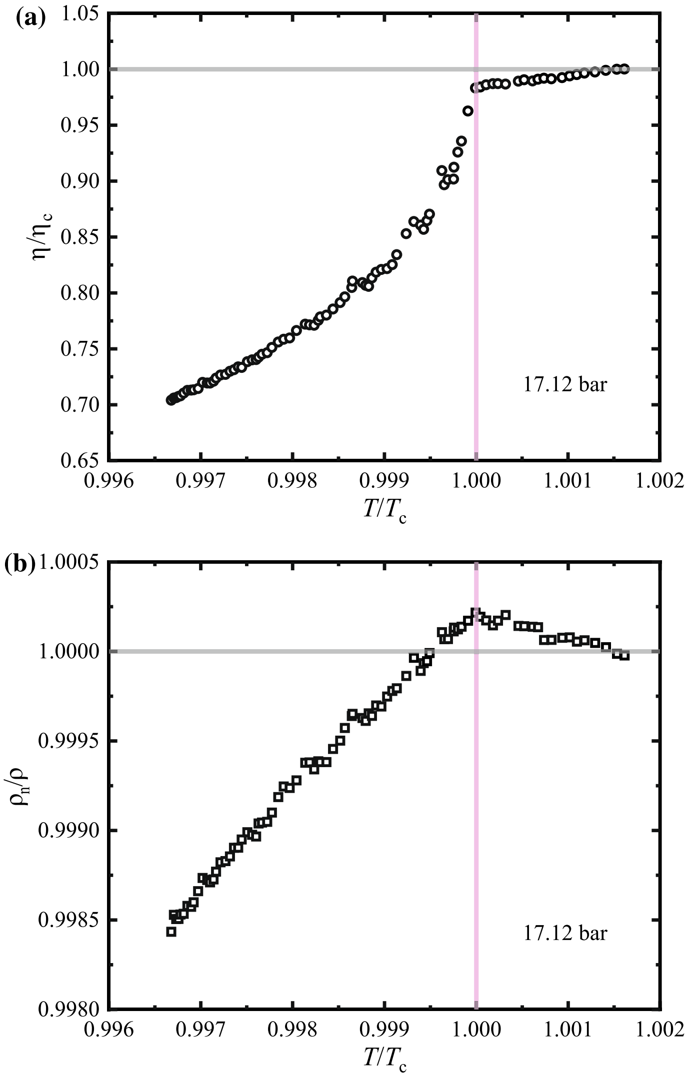

Fig. 2 **a** The normalized viscosity, η/η_c, vs the normalized temperature T/T_c, plotted against the reduced temperature T/T_c. T_c is marked by the vertical (lilac) line, while the horizontal grey line marks η_c. **b** The corresponding plot of ρ_n/ρ vs. T/T_c. Again the vertical (lilac) and horizontal (grey) lines mark T_c and ρ_n/ρ =1.0 (Color figure online)

which the experiment warmed by $\approx 2\ \mu$K for a temperature ramp of ~1 nK/s. One data set obtained in this fashion is shown in Fig. 2. The data emphasize the anomalous behavior. Immediately above T_c, the inferred viscosity shows a linear increase, while the inferred density decreased over the same temperature interval of $\approx 0.0015\ T_c$. We designate these quantities as “inferred" since the behavior was assumed to originate in the torsion head where the hydrodynamics could be simply written down. In fact, the origin of these effects is the viscous behavior of the fluid contained in the torsion rod.

The viscosity of ^{3}He decreases sharply at the superfluid transition with $\delta\eta/\eta_c \propto (1 - T/T_c)^{1/2}$ [2, 12, 13], where $\delta\eta = \eta_c - \eta(T)$ is the deviation of the viscosity from its value at T_c. This rapid variation of the viscosity near the transition temperature is likely responsible for the behavior discussed here.

2.3 Estimates

Our model takes into account the observation that as the system warms, the fluid in the head passes through T_c while the fluid in the torsion tube is still in the superfluid state. Assuming a linear warming rate, from Fig. 2a we estimate the normal fluid viscosity (in the tube) to be $\approx 0.75\ \eta_c$ as the fluid in the head passes through T_c. Over the course of the acquisition of the data discussed here, the normal–superfluid interface moves down the torsion tube. At the end of this time, both the fluid in the head and the tube are in the normal state, and the inferred viscosity and normal density attain their true normal fluid values.

The dimensions of the torsion tube were 3.81 mm long (l), 0.33 mm outer radius (r_o) and 0.25 mm inner radius (r_i). The fluid in the head of the torsion pendulum was contained in a disk-shaped cavity with radius 4.2 mm (r_h) and 95 μm height (d). The ratio of the inertia of the fluid in the rod to the fluid in the head (when both are fully locked) is expressed as $I_r/I_h = (r_i/r_h)^4 \times l/d = 5 \times 10^{-4}$. Since the torsion tube is fixed at the mounting point near the thermometer and cannot oscillate at that end, the maximum relative inertial contribution of the fluid in the torsion tube would be reduced by a factor 2 to 2.5×10^{-4}. This contribution is the same size as the anomaly seen in Fig. 2b. However, the viscous penetration depth is larger than the tube radius and the fluid in the torsion tube is in the locked regime. Thus we expect that the majority of the inertia of the fluid in the torsion tube is coupled to pendulum's motion over the whole temperature range studied in Fig. 2 and thus no changes in inertia would be visible. Additionally, a larger viscosity (attained as the fluid passes from the superfluid to the normal state) would lead to lower dissipation (consistent with the data shown in Fig. 2a) and a greater normal fluid coupling to the pendulum (the opposite of what is seen in Fig. 2b). Thus, for both reasons cited here, the viscous coupling of the inertial contribution cannot lead to the observed behavior.

Since the inertia of the fluid in the torsion rod cannot account for the observed behavior, we have to consider the increase in the viscosity. As we previously stated, an increase in the viscosity would be accompanied by a decreased dissipation, on account of the fluid in the torsion tube being in the well-locked regime. Beamish and co-workers [14] found that a portion of the apparent supersolid response in torsion pendulum experiments where the head and torsion tube were filled with solid ^{4}He could be attributed to the increase in the modulus of solid helium in the torsion rod [15]. Might the large viscosity of ^{3}He and the sharp increase in the viscosity near T_c produce a similar response? Liquids have no shear modulus, but near a glass transition, where the viscosity becomes large, there are measurable contributions (both real and imaginary) to an effective shear modulus [16]. If the fluid in the rod were replaced by solid helium, the shear modulus of the solid would contribute to the torsion constant of the torsion rod, raising the resonant frequency. Any increase in the elastic modulus of the solid would thus be interpreted as a lowered inertial contribution. Now consider a highly viscous fluid such as a glass that has very large viscosity. A high viscosity glass would contribute to the elastic modulus in the same way as a solid, with a higher viscosity more closely approximating solid behavior. However, the frequency of oscillation also comes into play. If the oscillation were at very low frequency, the liquid could not contribute significantly to the modulus, since liquids do not sustain shear. Thus, the concept of a storage modulus, $G_{storage} = \omega\eta$ is appropriate, and a fluid with higher viscosity would "stiffen" the torsion rod. Consequently, ΔG_{liquid}, the contribution to the effective torsion constant is related to the change in the viscosity ($\Delta\eta$) through $\Delta G_{liquid} = \Delta\eta\omega$ with ω being $2\pi f_0$ and f_0 being the resonant frequency. Rewriting equation 1 in [14] (and also see p 64 [17]),

$$\frac{\Delta f_{TOfluid}}{f_0} = \frac{1}{2}\frac{\Delta G_{liquid}}{G_{BeCu}}\frac{1}{\left(\frac{r_o}{r_i}\right)^4 - 1}, \tag{1}$$

$G_{BeCu} = 5.3\times10^8$ Pa is the torsion modulus for beryllium-copper. As seen in Fig. 2a, the rapid change in viscosity is of order 0.25 η_c and $\eta_c = 2.5\times10^{-2}$ Pa·s [11]. Thus $\Delta G_{liquid} = 36$ Pa and the fractional *increase* in frequency is estimated to be

$$\frac{\Delta f_{fluid\text{-}rod}}{f_0} = \frac{1}{2}\frac{36}{5.3\times10^8}\frac{1}{\left(\frac{0.33}{0.25}\right)^4 - 1} = 1.7\times10^{-8}. \tag{2}$$

The inertia of the normal fluid (ρ= 106.3 kgm^{-3}) in the torsion head is

$$I_f = \frac{1}{2}\pi\rho r_h^4 d = 5\times10^{-12}\ [\mathrm{kg\,m^2}], \tag{3}$$

and produces a maximum frequency shift of

$$\frac{\Delta f_{fluid\text{-}head}}{f_0} = \frac{1}{2}\frac{I_f}{I_{head}} = \frac{1}{2}\frac{5\times10^{-12}}{1.11\times10^{-8}} = 2.3\times10^{-4}. \tag{4}$$

corresponding to the frequency shift upon full loading by the normal fluid. $I_{head} = 1.1\times10^{-8}$ kg m^2. We can thus compute the relative shifts due to viscosity change in the torsion rod compared to the normal fluid in the head

$$\frac{\Delta f_{fluid\text{-}rod}}{\Delta f_{fluid\text{-}head}} = \frac{1.7\times10^{-8}}{2.3\times10^{-4}} = 0.74\times10^{-4}. \tag{5}$$

This would account for about a third of the observed anomalous decrease in ρ_n/ρ seen in Fig. 2 (lower). Since the estimate is for a liquid sample in the form of a rod in contact with an oscillating surface (the inner radius of the torsion tube) rather than a solid rod, the analogy of solid helium is inexact, because the angular displacement of all fluid elements at a given height in the torsion rod are not identical (the fluid is subject to radial shear as well as longitudinal shear); thus a greater viscosity would result in further radial locking of the fluid and an increased $G_{storage}$ contribution. We believe that the estimate we provide should be regarded as a lower limit and that the anomaly should be accounted for if a complete calculation were forthcoming (well beyond the scope of this paper).

We can apply the same principles to explore the observed behavior of η/η_c. Rewriting equation 4 in [14]

$$Q_{TO}^{-1} = \frac{1}{2}\frac{\Delta G_{liquid}}{G_{tube}}\frac{1}{\left(\frac{r_o}{r_i}\right)^4 - 1}Q_{He}^{-1} = \frac{1}{2}\frac{\Delta G}{G_{tube}}\frac{1}{\left(\frac{r_o}{r_i}\right)^4 - 1}\frac{G_{loss}}{G_{storage}}. \tag{6}$$

here G_{loss} is the "loss" modulus and $G_{storage}$ is the "storage" modulus in the complex response of the liquid to shear stress (See p 137 in [17]). Q_{TO}^{-1} corresponds to the dissipation observed in the torsion pendulum, while Q_{He}^{-1} characterizes the associated losses in the fluid contained in the rod. It is important to note that a Newtonian fluid would have an infinite loss tangent ($G_{loss} \gg G_{storage}$) and that $G_{loss} = G_{storage}$ marks the boundary between viscoelastic fluid ($G_{loss} \geq G_{storage}$), and gel-like behavior ($G_{loss} \leq G_{storage}$).

Unfortunately, there are no direct measurements of G_{loss}. We do however, have measurements of the change in drive voltage observed between the passage of T_c in the torsion pendulum head and the passage of T_c through the bottom of the torsion tube. The shift in drive voltage above T_c was found to be 0.0028 V (Fig. 3). This is to be compared to the drive voltage (0.0188 V) necessary to operate the empty pendulum (Q=7.5×10^5) at a constant amplitude. Thus we find Q_{TO}^{-1} = (0.0028/0.0188) × 1/Q = 2×10^{-7}. We remark that this is an exceedingly small dissipation. Using

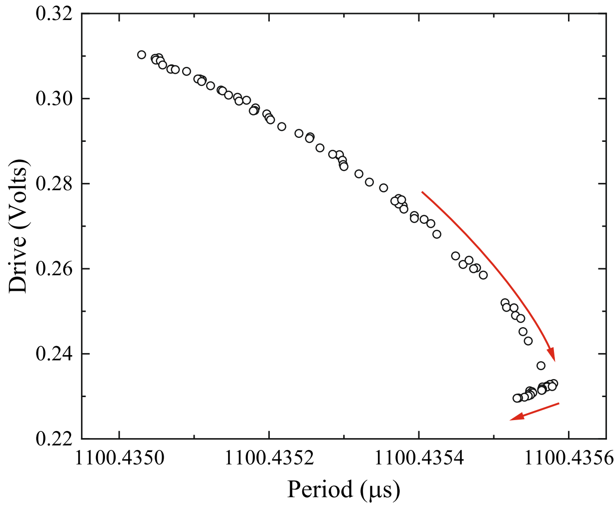

Fig. 3 The drive voltage plotted against the period shift in the vicinity of T_c. The top set of points corresponds to the data below T_c, while the data above T_c, corresponding to contributions attributed to the fluid in the torsion rod, appear as the lower set of data where the period decreases, together with decreased drive voltage. Arrows show the data progression as the fluid warms (Color figure online)

the previously calculated values for this torsion rod from Equation 2, we find $G_{loss}/G_{storage} = 2\times10^{-7}/6.8\times10^{-8} = 2.9$, giving a large loss tangent (but appreciably different from that of a Newtonian fluid or the crossover to the gel-like state).

3 Conclusions

We attribute the inferred changes in superfluid density and viscosity, observed decades ago in torsion oscillator experiments performed near the superfluid transition in ^{3}He to changes in the effective torsion modulus of the liquid ^{3}He in the torsion rod, in a manner related to viscoelastic phenomena or the mechanics of glasses. The large viscosity of the normal fluid and its rapid change contribute to the size of the signal. Other enabling factors that made these contributions visible were the excellent signal to noise and ability to ramp the temperature slowly. For later experiments, a reduction of this contribution was effected by decreasing the inner diameter of the torsion rod (as suggested in Reference [14]); thus most of these issues do not arise. However, few subsequent measurements have examined this region near T_c this closely, so similar phenomena could readily be missed. Additionally, separation of the bulk T_c from the suppressed T_c due to confinement or disorder would also allow the true fluid behavior under study to be measured. It is also possible that asymmetry in the torsion head might produce a small time-dependent pressure at the top of the torsion rod, and drive the fluid in the torsion rod, complicating interpretation.

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