Parametric amplification in a torsional microresonator

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We observe parametric amplification in a torsional micron-scale mechanical resonator. An applied voltage is used to make a dynamic change to the torsional spring constant. Oscillating the spring constant at twice the resonant frequency results in a phase dependent amplification of the resonant motion. Our results agree well with the theory of parametric amplification. By taking swept frequency measurements, we observe interesting structure in the resonant response curves. © *2000 American Institute of Physics.* [S0003-6951(00)05036-1]

Micromechanical torsional oscillators are highly useful tools in the physical sciences. The rotational motion makes these structures ideal for studies of magnetism and vortex physics.[1] They have also been used to detect very small strains, because the lever arm of the torsional motion increases the detection sensitivity. This has helped to gain understanding about inelastic processes in crystalline silicon[2] and silica glasses.[3] Ultrasmall resonators can detect small forces and have been shown to be very sensitive to electric charges.[4] The detection capabilities can be enhanced by taking advantage of dynamic behaviors that have not previously been observed in torsional oscillating systems. We describe a simple system in which the torsional spring constant of a micron-scale oscillator can be directly modified with an applied voltage. A modulation of the spring constant at twice the resonant frequency amplifies the resonant motion of the structure. This type of parametric amplification has been previously observed in cantilever oscillators.[5] These measurements were performed on extremely small resonators, with torsion members that have widths below 200 nm. The results shown, however, are applicable to structures on any size scale. Our measurements reveal interesting properties about the frequency response of parametrically amplified systems.

The expected behaviors of a mechanical parametric amplifier have been derived previously.[5,6] The predictions of this theory are presented here for later comparison to the actual data. Consider a system that is described by the following equation of motion:

$$I\ddot{\varphi}=-[\kappa-\kappa'(t)]\varphi-\frac{I\omega_0}{Q}\dot{\varphi}+\tau(\omega t), \tag{1}$$

where φ is angular displacement of the oscillator from equilibrium, κ is the time independent (normal) part of the torsional spring constant, and $\kappa'(t)$ is a dynamic modulation of the spring constant that is brought about by the application of a time dependent potential at exactly twice the driving frequency, 2ω. I is the moment of inertia, Q is the quality factor, ω_0 is the resonant frequency of the oscillator, and $\tau(\omega t)$ is the applied torque at frequency ω. If we drive the structure on resonance with an applied torque given by

$$\tau(t)=\tau_0\sin(\omega t+\theta), \tag{2}$$

with θ the phase between the ω and 2ω components with the result is a modulation (or time dependence) of the spring constant given by

$$\kappa'(t)=\kappa_0'\cos(2\omega_0 t). \tag{3}$$

Theory predicts that the system will respond with an amplitude of oscillation, φ_0 described by

$$\varphi_0=\frac{\tau_0 Q}{\kappa}\left[\frac{\cos^2\theta}{(1+Q\kappa_0'/2\kappa)^2}+\frac{\sin^2\theta}{(1-Q\kappa_0'/2\kappa)^2}\right]^{1/2}. \tag{4}$$

The first factor is the right hand side, $(\tau_0 Q)/(\kappa)$, is just the normal resonant response of an oscillator when $\kappa'(t)=0$, i.e., when no 2ω signal is applied. The second factor then acts as a phase-dependent gain. When the phase $\theta=0$, the system will be deamplified. When the phase $\theta=\pi/2$, the gain diverges to infinity when $\kappa_0'=(2\kappa)/(Q)$.

It is possible to fabricate a torsional microresonator that is well described by the earlier equations of motion. We have previously reported on the fabrication and measurement techniques used in the study of very small silicon oscillators.[7–9] Commercially available silicon on insulator wafers are patterned using electron beam lithography. The pattern is transferred into the top silicon surface using a reactive ion etch, and the small features are released in an isotropic wet etch that selectively attacks the buried oxide. We have studied the behavior of structures such as the one shown in Fig. 1. This consists of a rectangular paddle symmetrically suspended by narrow beams that are typically 150–200 nm in width. The length and width were varied over a large range. The minimum size of the paddles in which we observed parametric amplification was $2\times2\ \mu m^2$ and the largest was $4\times25\ \mu m^2$. All of the data in this letter is from a $4\times15\ \mu m^2$ paddle. Thin layers of Cr (5 nm) and Au (10 nm) are evaporated on the top surface and the underlying substrate so that electrical connection can be made. The structures to be tested are held under vacuum to avoid viscous damping effects. Motion is detected optically. Light from a He–Ne laser (632.8 nm) passes through a quartz window into the vacuum chamber and is focused onto the sur-

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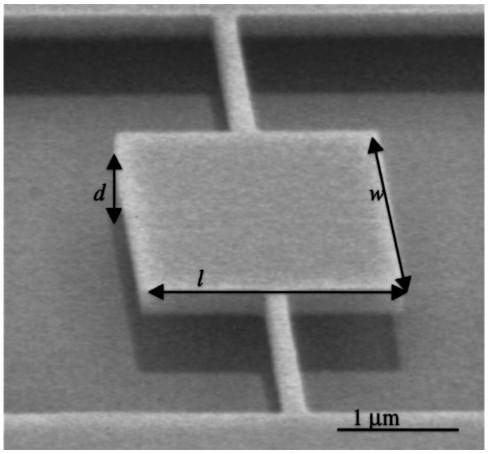

FIG. 1. A scanning electron micrograph of a torsional oscillator similar to the ones measured for this experiment is shown. The top silicon layer is 200 nm and the structure is suspended 400 nm above the underlying substrate. Thin layers of chrome and gold are deposited on the surface. The shadow from this evaporation can be seen beneath the oscillator.

face of the oscillator. Motion of the structure will result in a change in the reflectance due to interference, and this change is detected using an alternating current (ac)-coupled photoreceiver. The structures are driven electrostatically by applying a potential between the top and bottom surfaces. The substrate is grounded while the suspended structure is driven with an ac signal (with ω and 2ω components) that is added to a direct current (dc) voltage.

In earlier work, we have shown that the torsional mode can be excited in this fashion, even though the total torque on such a symmetric oscillator should be zero.[10] Slight asymmetries are incurred during the fabrication process and these are sufficient to allow the excitation of torsional motion. We have also shown that a small applied dc voltage will produce a substantial decrease in the effective spring constant, a property that enables parametric amplification of the motion. From these results, we know that if the structure is placed under a dc voltage, V_{dc}, which is added to an ac signal, $V_{\mathrm{ac}} \sin 2\omega t$, then the spring constant will be pumped by an amount

$$\kappa(t) = \frac{\epsilon_0 w l^3 V_{\mathrm{dc}} V_{\mathrm{ac}}}{12 d^3} \cos(2\omega t), \tag{5}$$

where ϵ_0 is the permittivity of free space, and l, w, and d are shown in Fig. 1. If we let $V' = (24\kappa d^3)/(Q\epsilon_0 w l^3 V_{\mathrm{dc}})$ and $A_0 = (\tau Q)/(\kappa)$, then using Eqs. (3), (4), and (5) we show that the resonant response as a function of the ac signal amplitude is

$$\varphi_0 = A_0 \left[\frac{\cos^2\theta}{(1 + V_{\mathrm{ac}}/V')^2} + \frac{\sin^2\theta}{(1 - V_{\mathrm{ac}}/V')^2} \right]^{1/2}. \tag{6}$$

To observe this effect, we drive the structures by applying a torque at a frequency ω and oscillate the spring constant at 2ω. The application of a voltage at 2ω will also produce a torque at that frequency. For an oscillator with a sufficiently high Q, this torque will not affect the response of the oscillator at ω. The optical response at the driving frequency is measured using a spectrum analyzer. The frequency, ω, is swept across a range that allows us to observe the resonant peak shape of the response.

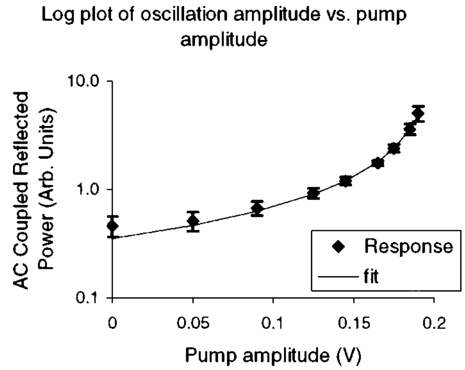

FIG. 2. The measured oscillation amplitude as a function of the amplitude of spring constant pump at twice the driving frequency. The asymptotic nature of this amplification is apparent. The free parameters of the fit were the amplitude at zero-gain and the asymptotic voltage, V'. The value of V' from the fit is 0.21 V, the expected value, based on the geometry of the oscillator and the measured frequency, is 0.20 +/−0.02 V. There are two significant sources of error. For the smaller signals, the noise from the electronics as well as the thermal motion of the oscillator is significant. For the larger signals where the gain is very high, there are fluctuations in the oscillation amplitude caused by noise in the amplifier electronics and the dc power supply. The asymptotic nature of the amplification causes these fluctuations to have a significant effect as the gain is increased.

Figure 2 shows the parametric amplification effect. A series of curves are shown in which the signal at the driving frequency is constant while the double frequency pumping signal varies over the range shown in the legend. The peak amplitudes as a function of the pumping signal are plotted and these values are fitted to a function of the form of Eq. (6). From the fit, we get a value of 0.20 +/−0.01 V for the asymptotic voltage V'. The predicted value is 0.20 +/−0.05 V.

These swept frequency measurements also show some behavior characteristics of parametric amplification that are not generally discussed. As the amplification increases, the peak width becomes substantially narrower and the peak assumes a form that is non-Lorentzian. This is due to the phase dependence of the amplification. The response to frequencies on either side of the resonance reflects the fact that the angular excursion and the pump (at 2ω) are of a different phase from that described in Eq. (4) and thus the amplitude at these frequencies will be deamplified. This is well illustrated in Fig. 3 in which several resonance curves are normalized to their respective peak values. The peak with the highest amplification has the narrowest width.

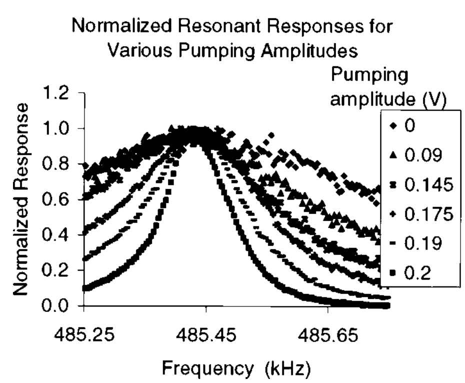

FIG. 3. The response curves for various pumping amplitudes are each normalized to their respective maxima to demonstrate the peak narrowing effect of the parametric amplification effect.

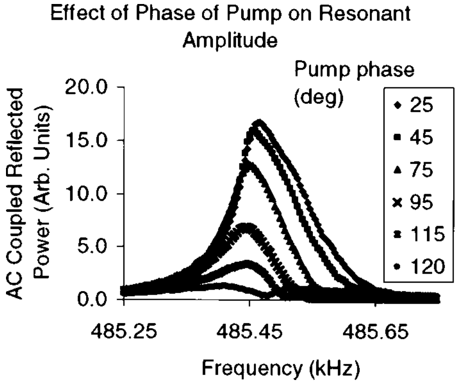

FIG. 4. Response curves for several different relative phases of the drive and the pump. A valley forms at the phase with the most deamplification.

Another interesting feature is shown in Fig. 4. We have varied the phase of the pump relative to the drive while the pump and drive amplitudes are both held constant. An arbitrary phase offset angle (120°) is incurred in the signal generation. As the phase is varied by 90°, we see the response evolve from the amplified state to the deamplified state. When the system is under deamplification, a dip in the response is observed at the resonant frequency. This is, again, a result of the phase dependence. The response is deamplified at the resonant frequency, but the sidebands are now amplified.

This amplification technique should allow a mechanical resonator to achieve a narrower bandwidth than it would be capable of in the unpumped state. The phase dependence of the amplification will lead to an increased sensitivity to phase noise. Our purpose in this letter is to demonstrate that parametric amplification can be realized in torsional systems. Because of their simplicity, these parametric amplification techniques could be extended to coupled mechanical oscillators whose coupling spring constant can be dynamically altered.[11] Electronic amplifiers of this type are well known for their low noise figure.

Parametric techniques will be critical in high frequency applications where the performance of mechanical devices become limited by processes such as thermoelastic and phonon–phonon effects that intrinsically limit the ωQ product.[12,13] We have demonstrated that torsional devices are capable of producing a response that is nearly an order of magnitude narrower than that achieved without parametric amplification.

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