ELSEVIER

Physica B 284–288 (2000) 293–294

PHYSICA B

www.elsevier.com/locate/physb

Flow of normal ^{3}He in aerogel

Dietrich Einzel[a,*], Jeevak M. Parpia[b]

[a] *Walther-Meissner-Institute, Walther-Meissner-Strasse 8, D-85748 Garching, Germany*
[b] *Department of Physics, Cornell University, Ithaca, NY 14853, USA*

Abstract

We consider the flow of normal ^{3}He in aerogel in the temperature regime $T_c \leqslant T \ll T_F$. A recent phenomenological treatment that predicted the transition from Drude to Hagen–Poiseuille flow at low T and low aerogel concentration, was restricted to small Knudsen numbers λ^*/d ($\lambda^* =$ quasiparticle mean free path of ^{3}He in aerogel, $d =$ sample dimension), the so-called slip regime. In this contribution we lift this restriction and present a variational calculation, valid at arbitrary Knudsen numbers. We calculate the slip length of liquid ^{3}He in aerogel and predict the temperature dependence of the flow resistance in this general situation.

Keywords: Aerogel; ^{3}He normal liquid; Transport theory

This paper aims at a description of flow of normal liquid ^{3}He in a channel oriented in the x-direction between two parallel plates at $z = \pm d/2$, filled with a dilute aerogel. We assume that the ^{3}He quasiparticles are characterized by an excitation spectrum ξ_k, a group velocity $\boldsymbol{v}_k$, an equilibrium Fermi distribution $n_k^0 = 1/[\exp(\xi_k/k_B T) + 1]$ with derivative $\varphi_k \equiv -\partial n_k^0/\partial \xi_k$, an inelastic temperature-dependent scattering rate $\Gamma^i(T) \propto T^2$. In addition, the presence of aerogel introduces an elastic scattering rate Γ^e, not present in ordinary Fermi liquids [1]. These processes result in an effective scattering rate $\Gamma^*(T) = 1/\tau^* = \Gamma^i(T) + \Gamma^e$ and a mean free path $\lambda_{kz}^*(T) = v_{kz}\tau^*(T)$, which is bounded from above at low T. The simplest form of the Landau–Boltzmann equation for the quasiparticle distribution function n_k in the stationary case for this flow geometry can be written as

$$\left(\frac{\partial}{\partial z} + \frac{1}{\lambda_{kz}^*}\right) n_k(z) = \frac{\varphi_k p_x}{\lambda_{kz}^*}\left[\frac{\tau^* F_x}{m} + a_1^\dagger u_x(z)\right]. \tag{1}$$

In Eq. (1) $u_x(z) = \rho^{-1}\sum_{k\sigma} p_x n_k$ is the velocity field, with ρ the total mass density, and F_x^{ext} the external driving force. The parameter $a_1^\dagger = \Gamma^i \tau^*$ describes the nonconservation of the mass current $g_x = \rho u_x$ in the presence of aerogel resulting from a conserving relaxation time approximation for the collision integral [2]. Introducing a dimensionless velocity field $w(z) = 1 + u_x(z)/u_0$ with $u_0 = \tau^* F_x^{\text{ext}}/m a_1^\dagger$ and an integral operator

$$K(z) = \delta(z) - a_1^\dagger \frac{\langle \varphi p_x^2 \lambda_z^{*-1} e^{-z/\lambda_z^*}\rangle_+}{2\langle \varphi p_x^2\rangle_+} \tag{2}$$

with $\langle A\rangle_+ \equiv \sum_{k\sigma} A_k \theta(v_{kz})$ (θ is the Heaviside step function), one may show that Eq. (1) is equivalent to the integral equation

$$Kw \equiv \int_{-d/2}^{d/2} dz'\, K(|z - z'|) w(z') = 1. \tag{3}$$

In deriving Eq. (3) we have used conditions for diffuse boundary scattering, i.e. $n_k^>(-d/2) = 0, v_{kz} > 0$ and $n_k^<(d/2) = 0, v_{kz} < 0$. Eq. (3) now serves as the starting point for a variational calculation of the averaged velocity field:

$$\langle u_x\rangle = u_0[(w,1) - 1] \tag{4}$$

with $(a,b) = d^{-1}\int_{-d/2}^{d/2} dz\, a(z) b(z)$, that should be stationary for an optimal choice for $w(z) \to v(z)$, which we assume to be of the general form

$$v(z) = v_0[\alpha_0 - h(z)]; \qquad u_x(z) \to u_x^{\text{var}}(z). \tag{5}$$

* Corresponding author.
E-mail address: einzel@badw.de (D. Einzel)

0921-4526/00/$ - see front matter
PII: S0921-4526(99)02628-9

$\langle u_x^{\rm var}\rangle$ is then optimized with respect to v_0, α_0:

$$v_0 = \frac{K_{h1} - (h,1)K_{11}}{K_{11}K_{hh} - K_{h1}^2}, \qquad \alpha_0 = \frac{K_{hh} - (h,1)K_{h1}}{K_{h1} - (h,1)K_{11}}, \tag{6}$$

where $K_{ab} = (a, Kb)$. Abbreviating $\gamma_0 \equiv \alpha_0 - v_0^{-1}$, we may write the velocity profile $u_x^{\rm var}(z)$ as

$$\frac{u_x^{\rm var}(z)}{u_0} = v_0\{\gamma_0 - h(z)\}$$

$$= v_0\gamma_0\left[1 - \frac{h(z)}{h(d/2) + \zeta_0^{\rm var}h'(d/2)}\right]. \tag{7}$$

An inspection of Eq. (7) shows that the length

$$\zeta_0^{\rm var} \equiv \left|\frac{u_x(d/2)}{u'_x(d/2)}\right| = \frac{\gamma_0 - h(d/2)}{h'(d/2)} \tag{8}$$

can be interpreted as a generalization of the velocity slip length, defined by $u_x^{\rm var}(z = -\zeta_0) = 0$ to arbitrary Knudsen numbers. The averaged velocity field reads finally

$$\frac{\langle u_x^{\rm var}\rangle}{u_0} = v_0\gamma_0\left[1 - \frac{(h,1)}{h(d/2) + \zeta_0^{\rm var}h'(d/2)}\right]. \tag{9}$$

We now specify the profile $h(z)$ to be

$$h(z) = \cosh qz, \qquad qv_{\rm F}\tau^* = \sqrt{5/a_1^*}. \tag{10}$$

This choice is motivated by the result obtained in Ref. [3, see Eq. (18) and Fig. 2]. In the slip regime $\lambda_{kz}^*/d \ll 1$, the slip length ζ_0^* is controlled by elastic scattering off the aerogel strands at low T:

$$\lim_{\lambda_{kz}^*/d\to 0} \zeta_0^{\rm var} = \zeta_0^* = \frac{8}{15}v_{\rm F}\tau^*. \tag{11}$$

Expressing $h(z)$ in form (10), the velocity profile (7) becomes

$$\frac{u_x^{\rm var}(z)}{u_0} = \frac{a_1^*}{1 - a_1^*}\left[1 - \frac{\cosh(qz)/\cosh(qd/2)}{1 + q\zeta_0^*\tanh(qd/2)}\right]. \tag{12}$$

For the cross-sectional average $\langle u_x^{\rm var}\rangle$ we obtain in the same limit

$$\frac{\langle u_x^{\rm var}\rangle}{u_0} = \frac{a_1^*}{1 - a_1^*}\left[1 - \frac{2}{qd}\frac{\tanh(qd/2)}{1 + q\zeta_0^*\tanh(qd/2)}\right]. \tag{13}$$

Eq. (12) describes the expected transition from Hagen–Poiseuille flow ($qd \ll 1$, *parabolic* profile for $u_x^{\rm var}(z)$) to Drude flow ($qd \gg 1$, *flat* profile for $u_x^{\rm var}(z)$) for low-density aerogel. We have thus derived and hence justified the result obtained from the phenomenological treatment of Ref. [3] (cf. Eq. (17)) from microscopic theory. In contrast to the phenomenological treatment, where the slip length ζ_0 entered as an unknown parameter, our microscopic treatment yields the theoretical estimates (8) and (11) for it, which corresponds to the "lower bound" estimate derived in Ref. [4] in the absence of elastic scattering. The evaluation of our results (7) and (9) for arbitrary Knudsen numbers, as well as the discussion of the influence of the Fermi liquid interaction and the effects of finite frequencies will be discussed in Ref. [2].

Acknowledgements

Work was supported by NATO (CRG960127) and NSF (DMR-9705295).

References

[1] D. Rainer, J. Sauls, J. Low Temp. Phys. 110 (1998) 525.
[2] D. Einzel, J. Parpia, to be published.
[3] D. Einzel, J. Parpia, Phys. Rev. Lett. 81 (1998) 3896.
[4] H. Jensen et al., J. Low Temp. Phys. 41 (1980) 473.