Finite-Size Effects and Shear Viscosity in Superfluid ^{3}He-*B*

Dietrich Einzel
Walther Meissner Institut für Tieftemperaturforschung, D-8046 Garching, West Germany

and

Jeevak M. Parpia[a]
Texas A&M University, College Station, Texas 77843
(Received 12 August 1986)

The effective viscosity of ^{3}He-*B* has been measured by means of a torsional pendulum with a characteristic dimension of 135 μm. A comparison between the experimental results and theory—including the effects of (normal) fluid slip, Knudsen flow, and Andreev scattering of quasiparticles from spatial variations of the order parameter—reveals that while the experimental results at high pressure are bounded by the theoretical extremes considered, the low-pressure data are not; in fact, the disparity is seen to increase at lower pressures and lower temperatures.

PACS numbers: 67.50.Dg, 47.45.Gx, 67.20.+k, 67.50.Fi

The interrelation of the geometry of a sample and the transport coefficient measured in an experiment is well known from the early studies of Knudsen[1] of rarified gases and from the result of de Haas and Biermasz on the thermal conductivity of insulators.[2] In both of these experiments, the finite size of the cell geometry affected the measured transport parameter when the mean free path approached the size of the container. Analogous experiments have been carried out in ^{4}He by Whitworth[3] and Greywall,[4] and in ^{3}He by Eisenstein, Swift, and Packard[5] and Parpia and Rhodes.[6] The ^{3}He experiments were performed at 0 bar in the normal Fermi-liquid region and revealed that, unlike the experiments performed with classical particles or phonons, there is a discrepancy between models of hydrodynamic slip for diffuse scattering of quasiparticle excitations from surfaces,[7,8] and these experiments.

In this Letter we report results of a systematic study of the pressure and temperature dependence of the measured effective shear viscosity of the normal fraction of ^{3}He-*B*, relative to its value at the superfluid transition, $(\eta/\eta_c)_{\rm eff}$, together with theoretical estimates for that quantity. The study extends over a pressure range of 0 to 29 bars and a temperature range down to $T/T_c = 0.25$.

The systematic variation of the temperature and pressure permits us to study the hydrodynamic properties of quasiparticles scattering from surfaces over a certain range of the Knudsen number $N_{\rm Kn} = l_\eta/d$, where l_η is the viscous mean free path and d is the characteristic cell size. The viscous mean free path in the normal Fermi liquid $l_\eta^N = v_{\rm F}\tau_\eta$, with $v_{\rm F}$ the Fermi velocity and τ_η the viscous relaxation time, behaves as T^{-2}, thus precluding the systematic study of effects in the large-Knudsen-number regime except at low pressures. The mean free path l_η^S of thermal excitations from superfluid ^{3}He-*B* (the so-called Bogoliubov quasiparticles), on the other hand, varies exponentially $\propto \exp(\Delta/kT)$ (Δ is the isotropic gap) at low temperature. The Knudsen number (for a cell size of about 100 μm) therefore ranges from 0.02 at T_c (29 bars) to about 20 at the lowest temperature (0 bar) and thus allows for the observation of the Knudsen transition, $N_{\rm Kn} \gtrsim 1$ at all pressures.

The motivation of this research was to use the techniques developed to describe strongly nonhydrodynamic behavior in the Knudsen regime,[7] as well as through "quantum-slip" effects,[8] to expose discrepancies between the calculated and measured effective viscosities as they develop with pressure and temperature.

The experiments were carried out with a torsional oscillator having a disk-shaped ^{3}He space with a characteristic height of 135 μm. The oscillator was operated at its resonant frequency of $\sim$385 Hz and maintained at a constant amplitude of oscillation ($\sim$10 Å) by a phase-locked loop. The effective viscosity and superfluid density were obtained by simultaneous measurement of the dissipation and period of oscillation, and included corrections for the torsion rod itself. The oscillator, demagnetization stage, and cryostat as well as the hydrodynamics have been described elsewhere.[9]

Above T_c in the Fermi-liquid region, the finite-size modifications to the shear viscosity are well described by a slip correction.[7] In this case, the effective viscosity is modified from that in the bulk through the relation

$$1/\eta_{\rm eff} = 1/\eta_{\rm bulk}(1+6\zeta/d) = 1/\eta_{\rm bulk}(1+a_s l_\eta/d). \qquad (1)$$

Here ζ is the so-called slip length, i.e., a characteristic length at which the velocity field of the fluid extrapolates to zero behind the wall. It is related to the viscous mean free path l_η through a slip coefficient a_s. The slip approximation forms a first-order correction to the usual hydrodynamic description and should be applicable in low-Knudsen-number regime.

In normal ^{3}He the slip coefficient a_s has been measured by two groups,[5,6] who found this quantity at low pressure to be smaller than the theoretical value[7] for diffuse quasiparticle scattering $a_s = \text{const} = \frac{139}{40}$. In addition, Parpia and Rhodes[6] observed the Knudsen minimum at a Knudsen number of 1.33 rather than 2 as predicted by theory.

These low-pressure results have been confirmed by the experiments reported in this Letter. The experimentally determined slip coefficient a_s reveals a pronounced pressure dependence which is exposed in Fig. 1 (circles), together with the pressure-independent theoretical prediction for diffuse scattering of Jensen *et al.*[7] (dashed line). The results are in good agreement with theory at high pressures but show a significant deviation at lower pressures. This deviation has a sign which is inconsistent with any admixture of specular scattering. Therefore, we have not considered this possibility in the analysis to follow.

In the superfluid B phase, the validity of the slip approximation breaks down very quickly with lowering temperature at all pressures. For a comparison of experiment and theory the slip concept has to be generalized therefore by the introduction of a temperature- (and pressure-) dependent Knudsen function $S(T)$ into the law of Hagen and Poiseuille, which relates the averaged (normal) velocity field $\langle u \rangle$ to an external driving force F^{ext} via

$$\langle u \rangle = d^2 \rho_n S(T) F^{\text{ext}} / 12 \eta_{\text{bulk}} m. \tag{2}$$

Here m is the mass of the ^{3}He atom and ρ_n the normal-fluid density. The velocity field is obtained from the variational solution of a scalar Boltzmann equation for Bogoliubov quasiparticles, which is specified by a well

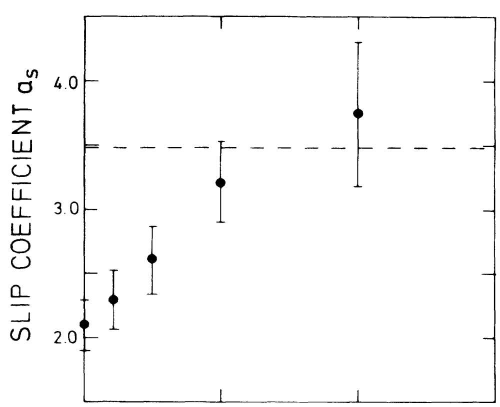

FIG. 1. The pressure dependence of the experimentally determined slip coefficient, a_s, defined as the ratio of slip length and viscous mean free path in normal ^{3}He (circles). Also shown is the pressure-independent theoretical result of Jensen *et al.* (Ref. 7) for diffuse quasiparticle scattering from the walls (dashed line).

controlled relaxation-time approximation for the collision integral and amended by a general boundary condition for the quasiparticle distribution function at the wall (for details see, e.g., Refs. 7 and 8). At low Knudsen numbers $S(T)$ reduces to the simple slip correction $S(T) = 1 + 6\zeta/d$ for the parallel plate geometry, whereas in the Knudsen limit $S(T)$ is seen to diverge logarithmically $\propto \ln(l_\eta/d)$. It should be emphasized that Eq. (2) is valid for arbitrary Knudsen numbers and a general class of laws for the scattering of quasiparticles from the wall. For a comparison of experiment with theory one needs the bulk viscosity η_{bulk},[10,11] and the Knudsen function $S(T)$,[8] and can then make use of the relationship

$$(\eta/\eta_c)_{\text{eff}} = (\eta/\eta_c)_{\text{bulk}} S(T_c)/S(T). \tag{3}$$

Einzel *et al.*[8] have made such an analysis of the high-pressure viscosity data of Archie *et al.*[12] taken in the superfluid B phase. In this experiment, a pronounced droop of the experimental viscosity away from the expected bulk result was observed. In order to explain the difference between $(\eta/\eta_c)_{\text{eff}}$ and $(\eta/\eta_c)_{\text{bulk}}$ at temperatures below $T/T_c = 0.6$, slip theory had to be modified to include the effects both of a transition to Knudsen flow and of quantum mechanical reflection of Bogoliubov quasiparticles from spatial variations of the order-parameter components near the container surfaces, the so-called Andreev reflection.[13] Andreev scattering involves the conversion of a particlelike into a holelike excitation near the surface due to a (partial) suppression of the gap. The reflected quasihole has all components of the group velocity reversed but a nearly unchanged momentum. Therefore, with decreasing temperature there is less exchange of transverse momentum between the quasiparticle gas and the wall compared to that for purely diffuse scattering, resulting in an enhanced slip length (quantum slip effect) or, more generally, an enhanced Knudsen correction $S(T)$.

The droop away from the bulk viscosity could be accounted for by the assumption of a "superposition" of diffuse and Andreev scattering of quasiparticles from the surfaces at the border of the Knudsen transition, by use of a smooth model order-parameter suppression profile.

In what follows, however, we consider the consequences of Andreev scattering from a steplike order-parameter variation only. In this case the energy dependence of the reflection amplitude is known[13] to be $R(E) = [E - (E^2 - \Delta^2)^{1/2}]/[E^2 + (E^2 - \Delta^2)^{1/2}]$ and to overestimate the influence of Andreev scattering.

The data for $(\eta/\eta_c)_{\text{eff}}$ at four different pressures are plotted in Fig. 2 as circles, together with $(\eta/\eta_c)_{\text{bulk}}$ (dashed line) as a function of inverse reduced temperature and can be read off against the left scale. For comparison purposes we show the theoretical results of the model with a diffuse-scattering boundary condition (solid lines 1) and that including in addition Andreev scattering from a step order-parameter discontinuity (solid lines

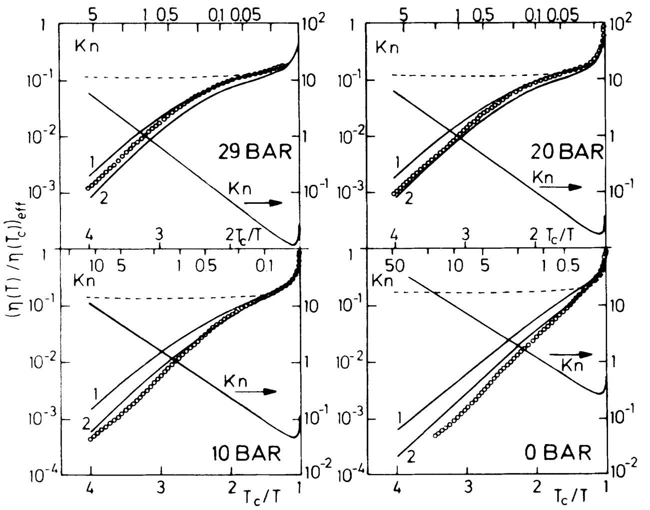

FIG. 2. The effective viscosity of superfluid ^{3}He-B (left scale) at four different pressures as a function of inverse reduced temperature. The experimental data from the 135-μm torsional oscillator are shown as circles, and the bulk theoretical result as dashed lines. Two theoretical estimates for the effective viscosity are represented by the solid lines 1 (diffuse scattering) and 2 (diffuse plus Andreev scattering from a steplike order-parameter profile). Also shown as solid lines is the temperature variation of the Knudsen numbers at the four pressures (right scale, see also upper scales).

2). Also illustrated are the Knudsen numbers N_{Kn} at the four pressures and for the 135-μm spacing (right-hand scale).

It should be emphasized that one might expect the experimental data to lie between the theoretical curves 1 [corresponding to a "minimum" for $S(T)$ because of the diffuse scattering assumption] and 2 [corresponding to a "maximum" for $S(T)$ due to the overestimation of the Andreev reflection amplitude].

First of all, the experimental data for $(\eta/\eta_c)_{\text{eff}}$ are seen to decrease monotonically with decreasing temperature at all pressures. At the low-temperature end the effective viscosity increases, as expected, monotonically with increasing pressure, because $S(T)$ scales roughly with $1/\tau_N(T_c) \propto T_c^2$. Above a narrow regime of crossover temperatures centered at $T/T_c \cong 0.7$, this pressure dependence is inverted, i.e., $(\eta/\eta_c)_{\text{eff}}$ decreases with increasing pressure and exceeds the bulk value $(\eta/\eta_c)_{\text{bulk}}$, an effect which is particularly pronounced at 0 bar. This nonintuitive result can be understood by our noting that the Knudsen-function ratio $S(T)/S(T_c)$ drops rapidly below T_c by up to a factor of 2.5, because of the change in the quasiparticle group velocity, in a manner similar to the Knudsen number N_{Kn}, plotted in Fig. 2. This drop is most pronounced at the lowest pressure, where the effective viscosity attains the highest values. Overall, the effective viscosity is closest to the bulk one at 29 bars and shows the greatest size effects at 0 bar.

It is evident that there is a remarkable qualitative agreement between experiment and theory. At high pressures (20 and 29 bars) there is also excellent quantitative agreement in the sense that the purely diffuse (curve 1) and diffuse-plus Andreev-scattering (curve 2) boundary conditions lead to minimum and maximum finite-size corrections, respectively, the data ranging somewhere between these two limits. At low pressures (0 and 10 bars), however, the low-temperature data points are seen to fall even below the theoretical curves 2.

Because of possible discrepancies in the temperature scales used ($\sim$5%), a detailed analysis of the deviation is not possible. However, it is unlikely that the discrepancies are due to effects of the temperature scale alone, since a reduction of the deviation at low pressure would increase the deviation at high pressures.

In Table I we list the parameters used to generate the theoretical curves. Fermi velocities were calculated from Greywall's data.[15] The quasiparticle lifetime at the transition $\tau_N(T_c)$ is poorly known experimentally at low pressures as is the scattering parameter λ_2, which specifies the viscous relaxation time of our model. We therefore used values from Wheatley[16] and theoretical estimates from Pfitzner.[17] It turns out, however, that the results are not very sensitive to the choice of λ_2 at low temperatures. The zero-temperature gap is the dominant parameter that enters into the determination of the Knudsen number at low temperatures through the ex-

TABLE I. Pressure dependence of the theoretical input parameters that enter into the calculation of the viscous mean free path.

Pressure	$\Delta(0)/kT_c$[a]	m^*/m[b]	$10^8\tau_N(T_c)$[c] (sec)	λ_2[d]
0	1.73	2.76	50	0.68
10	1.85	3.81	12	0.74
20	2.00	4.63	5.4	0.75
29	2.10	5.76	4.0	0.74

[a]Reference 14.
[b]Reference 15.
[c]References 16 and 17.
[d]Reference 17.

ponential growth of the mean free path; we used experimental values from Bloyet *et al.*[14]

In summary, we have presented the results of a systematic survey of the effects of Knudsen flow in superfluid ^{3}He-B. The results show that large reductions (upon the order of 3 orders of magnitude) are evident in the measured effective viscosity. The general shape of these modifications is matched by theoretical calculations of the viscosity in a finite-geometry system. It is also evident that in addition to the usual diffuse scattering of quasiparticles at surfaces, effects due to Andreev reflection processes can successfully account for the finite-size modifications of the bulk viscosity at high pressure. At low pressure, the experimental effective viscosity can be qualitatively but not quantitatively understood within the assumptions of our theoretical model. The remaining discrepancies do not arise solely at high values of the Knudsen number but rather appear to be pressure dependent. This would imply the need to introduce and physically interpret an additional pressure- (and, in the superfluid, temperature-) dependent factor relating the slip length to the viscous mean free path, or some pressure dependence of the epoxy-^{3}He interface's scattering properties. Perhaps the nature and thickness of the highly localized surface layers play a role in the scattering of quasiparticles. We therefore propose to perform experiments on surfaces of well characterized roughness, to attempt to determine the origin of these pressure-dependent effects.

On the theoretical side, we believe that the relaxation-time approximation for the quasiparticle collision integral and the variational procedure for the solution of the integral equations for the velocity profile are well controlled elements of our model. The description of surface scattering, on the other hand, might need some improvement. The suppression of the order parameter, for example, is not calculated self-consistently; inelastic scattering processes, finite–sticking-time effects, and the possibility of bound quasiparticle states in the surface layer have been ignored. The consequences of these and similar microscopic effects on the effective viscosity are presently unknown and should be a matter of further theoretical investigation.

We are indebted to Peter Wölfle for his assistance in all phases of this work, as well as to Girgl Eska, Peter Hirschfeld, and Vincent Kotsubo for their comments. This research was supported by the National Science Foundation through Grant No. DMR-821829 and by a NATO collaborative grant. One of us (J.M.P.) wishes to acknowledge support by the Alfred P. Sloan Foundation as well as the hospitality of the Walther Meissner Institut and the Technische Universität München, theoretical physics group, during a visit.

[a]Present address: Physics Department, Cornell University, Ithaca, NY 14853.

[1]M. Knudsen, *The Kinetic Theory of Gases* (Methuen, London, 1950).

[2]W. J. De Haas and Th. Biermasz, Physica (Utrecht) **2**, 673 (1935).

[3]R. W. Whitworth, Proc. Roy. Soc. London, Ser. A **244**, 390 (1958).

[4]D. Greywall, Phys. Rev. B **23**, 2152 (1981).

[5]J. P. Eisenstein, G. W. Swift, and R. E. Packard, Phys. Rev. Lett. **45**, 1199 (1980).

[6]J. M. Parpia and T. L. Rhodes, Phys. Rev. Lett. **51**, 805 (1983).

[7]H. Jensen, H. Smith, P. Wölfle, K. Nagai, and T. M. Bisgaard, J. Low Temp. Phys. **41**, 473 (1980).

[8]D. Einzel, P. Wölfle, H. Højgaard Jensen, and H. Smith, Phys. Rev. Lett. **52**, 1705 (1984).

[9]J. M. Parpia, D. G. Wildes, J. Saunders, E. K. Zeise, J. D. Reppy, and R. C. Richardson, J. Low Temp. Phys. **61**, 337 (1985).

[10]J. Hara, Y. A. Ono, K. Nagai, and K. Kawamura, J. Low Temp. Phys. **39**, 608 (1980).

[11]D. Einzel, J. Low Temp. Phys. **54**, 427 (1984).

[12]C. N. Archie, T. A. Alvesalo, J. D. Reppy, and R. C. Richardson, J. Low Temp. Phys. **42**, 1076 (1980).

[13]A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].

[14]D. Bloyet, E. Varoquaux, C. Vibet, O. Avenel, P. M. Berglund, and R. Combescot, Phys. Rev. Lett. **42**, 1158 (1979).

[15]D. Greywall, Phys. Rev. B **27**, 2747 (1983).

[16]J. C. Wheatley, in *Progress in Low Temperature Physics* (North-Holland, Amsterdam, 1978), Vol. 7, p. 1.

[17]M. Pfitzner, Ph.D. thesis, Technische Universität München, 1985 (unpublished).