First Observation of the Knudsen Minimum in Normal Liquid ^{3}He

J. M. Parpia and T. L. Rhodes
Department of Physics, Texas A & M University, College Station, Texas 77843
(Received 8 July 1983)

The viscosity of liquid ^{3}He contained in a 45-μm-high chamber has been studied. The slip correction and a clearly defined maximum in the effective viscosity are resolved in this experiment. The magnitude of the slip correction and the position of the maximum in the effective viscosity are found to be in disagreement with recent theoretical work.

PACS numbers: 67.50.Dg, 47.45.Gx, 67.40.Hf

In the course of the past five years, there has been a growing realization that finite quasiparticle mean-free-path effects modify and eventually dominate measurements of the viscosity of liquid ^{3}He at low temperatures. This Letter describes an experiment which for the first time observes the Knudsen minimum in the inverse effective viscosity for normal liquid ^{3}He. The results have sufficient resolution and accuracy to allow a critical comparison to recent theoretical work and expose quantitative discrepancies between theory and experiment.

The theoretical results of Jaffe[1] and Jensen *et al.*[2] were able to predict two distinct and separable effects of the finite quasiparticle mean free path in liquid ^{3}He. The first effect, slip at the boundary between liquid ^{3}He and the walls of the rheometric device, leads to a temperature-independent offset in the effective viscosity measured in the course of an experiment. This effect was observed in experiments carried out at Berkeley.[3] However, the relatively open geometries of the tubes (diameter $d > 250$ μm) precluded the observation of further modifications to the viscosity due to long mean free paths. We have constructed a torsion pendulum experiment with a dimension sufficiently small so as to resolve the departure of the measured effective viscosity from the bulk temperature dependence, and to allow the observation of the Knudsen minimum in normal liquid ^{3}He.

The data reported in this Letter were obtained with use of an Andronikashvili-type torsional oscillator with a resonant frequency of ~ 3525 Hz. The liquid sample was contained in a cavity of radius 0.533 cm and a height determined experimentally to be 45.3 ± 3.0 μm. A hollow beryllium-copper torsion rod connected the liquid sample to the thermometer and to the refrigeration cell which was cooled by a $PrNi_5$ nuclear demagnetization stage.[4] The thermometry agent was lanthanum-diluted cerium magnesium nitrate (LCMN) with a dilution ratio of 19:1 calibrated against a melting-curve thermometer[5] mounted on the exterior of the sample chamber. The calibration used the P-T relation measured by Halperin *et al.*[6] and Greywall and Busch,[5] and ranged from 45 to 5.9 mK as well as at the values of $T_A = 2.752$ mK, $T_B = 2.179$ mK, and $T_N = 1.100$ mK. On this temperature scale the superfluid transition, T_c, at saturated vapor pressure is found to be 1.13 mK. Thermal hysteresis between the melting-curve thermometer and the LCMN thermometer was found to be less than 0.2 μK at T_c, and the calculated thermal gradient was less than 10 μK at T_c.

The drive voltage necessary to maintain the oscillator at a constant amplitude of motion was measured, and by means of ringdown measurements, converted to a Q value. The resonant period was measured simultaneously and allowed a precise calibration of the entrained mass of the fluid via the following procedure. The empty oscillator was cooled to 1 mK. The apparatus was then warmed to 1.5 K, the oscillator filled with ^{3}He and pumped continuously with a charcoal dipstick to maintain a pressure of < 0.01 bar, and cooled to T_c. At this temperature the ^{3}He is essentially locked to the cell as a result of the large viscous penetration depth. The shift in the period of the oscillator from its value when full and when run empty was determined and converted to the absolute moment of inertia of the liquid (I_f) through the known moment of inertia of the epoxy torsion head (I_s). By further demagnetization of the magnetic refrigerator, the liquid ^{3}He was cooled deep into the superfluid B phase to a temperature of 0.4 mK. The period shift due to the progressive decoupling of the superfluid component was measured and extrapolated to its zero-temperature value where all the fluid would be at rest relative to the viscometer. These period shifts yielded values of the cell height of 46.8 and 43.8 μm, respectively.

A check of the cell height was accomplished by using the known hydrodynamics of ^{3}He in an Andronikashvili oscillator.[7] As a result of the large penetration depth, the dissipation due to the ^{3}He follows the relation

$$Q^{-1} = (I_f/I_s)d^2\rho\omega/12\eta, \quad (1)$$

where η is the viscosity, ρ the density of the liquid, and ω the angular frequency of oscillation. A fit to the measured dissipation due to the liquid between 4.5 mK $\leq T \leq$ 10 mK was determined to be $Q^{-1} = 4.518\times10^{-7}T^2 + 1.987\times10^{-6}$. Equation (1) can be inverted and by use of the arithmetic mean of the two cell-height calibrations, $d = 45.3$ μm, and the slope of the Q^{-1} vs T^2 dependence, the coefficient for the bulk viscosity ηT^2 determined to be 2.62 P mK^2 which is in close agreement with previous experiments[3,8-10] at this pressure. These measurements are relative to the moment of inertia of the torsion head which was found to be (0.1233 g cm^2)$\pm$ 5%. The uncertainty is smaller than that of previous experiments[8] as care was taken to weigh the epoxy-head parts during assembly. In addition to verifying that the cell dimension is consistent with earlier measurements of the viscosity at 0 bar this check confirms the accuracy of the temperature scale.

A portion of the results between 7.7 and 1.13 mK are displayed in Fig. 1. The drive voltage is plotted against the square of the temperature for both the full and the empty cell. The dissipation (Q^{-1}) is also displayed for reference on the right-hand side. The damping due to the empty cell is subtracted from the full-cell data and the corresponding Q^{-1} due to the entrained liquid computed. It is evident that the dissipation in the linear region has a temperature-independent offset which can be related to the slip at the boundaries of the oscillator. In order to facilitate comparison to the theoretical results of Jensen *et al.*,[2] the dissipation and temperature scales are converted to dimensionless parameters.

The slip theory of Jensen *et al.*[2] replaces the viscosity in Eq. (1) with an effective viscosity defined by

$$\frac{1}{\eta_{\rm eff}} = \left(\frac{1}{\frac{1}{5}np_{\rm F}d}\right)\left(\frac{T^2}{T_d^2} + \frac{139}{40}\right), \quad (2)$$

where n is the number density, $p_{\rm F}$ the Fermi momentum, and T_d the temperature at which the mean free path, λ, is equal to the cell height, d. The first term is precisely the inverse of the bulk viscosity and the second term the temperature-independent slip parameter. We calculate $T_d^2 = 2.13$ mK^2, and by inverting Eq. (1), inserting $\eta_{\rm eff}$ for η, and substituting for the fit for Q^{-1}, determine for the normalized inverse effective viscosity the expression

$$(\tfrac{1}{5}np_{\rm F}d)/\eta_{\rm eff} = 0.9992\,T^2/T_d^2 + 2.055$$

which is to be compared with the theoretical value

$$(\tfrac{1}{5}np_{\rm F}d)/\eta_{\rm eff} = T^2/T_d^2 + 3.475.$$

The normalized inverse effective viscosity is plotted against the reduced temperature T^2/T_d^2 in Fig. 2. The temperature scale is identical to the ratio d/λ, the Knudsen number. This figure

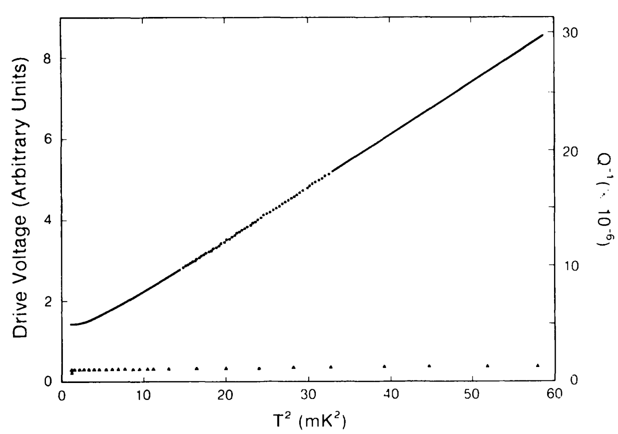

FIG. 1. The drive voltage and the corresponding Q^{-1} at saturated vapor pressure (circles) and for the empty cell (triangles) as a function of the square of the temperature.

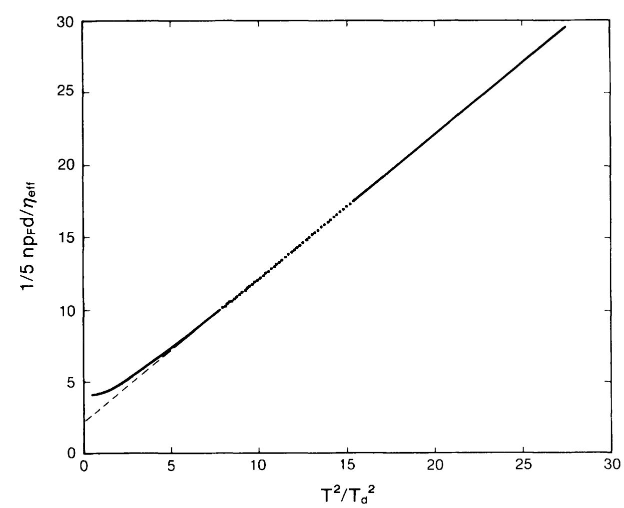

FIG. 2. The normalized inverse effective viscosity plotted against the square of the reduced temperature. The abscissa also corresponds to the Knudsen number. The theory, modified to reflect the smaller slip parameter, is plotted as a dashed line.

illustrates the experimentally determined slip "correction" shown as the dashed line, as well as the progressive departure from the slip theory for Knudsen numbers $K \leq 10$. The qualitative explanation for this behavior resides in the assumptions of the slip theory which allows the velocity of the fluid at the boundary to be different from that of the walls, invoking a boundary layer in which the velocity profile in the fluid approaches that predicted by Poiseuille flow, and defines a slip length where the extrapolated velocity would go to zero. The slip length varies as the mean free path, resulting in a temperature-independent slip parameter. As the Knudsen number decreases, the assumptions of the slip theory break down as the "bulk" region where Poiseuille flow occurs shrinks and the "boundary layer" expands. The velocity profile and the consequent effective viscosity have to be calculated numerically. It is clear that the theoretical value of the slip parameter is larger (3.475) than the experimental value of 2.055. The theory explicitly assumes diffuse scattering of quasiparticles and can allow for specular scattering, which unfortunately only serves to enlarge the discrepancy. An examination of results of the Berkeley[3] group reveals a discrepancy of similar magnitude for their smallest tube size where the slip parameter is found to be 69% ± 5.5% of the theoretical value as compared to our value of 59% ± 7%. It should be noted that their results tend toward the theoretical value with increasing tube size. The smaller slip parameter measured in both the experiments implies that additional momentum transfer occurs in the boundary layer. The explanation may be in the nature of the surface, or the substrate material (epoxy 1266) used in both the experiments.

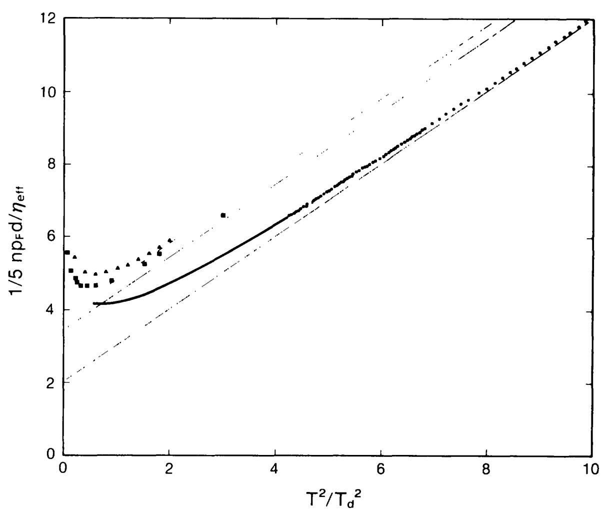

FIG. 3. A comparison of the experimental data in the Knudsen regime (circles) to results of Jaffe (triangles) and Jensen *et al.* (squares). The dashed lines represent the slip model as calculated by Jaffe (short-dashed), Jensen *et al.* (short-dash–long-dashed), and as determined by this experiment (long-dashed). The finite intercept of the various dashed lines corresponds to the slip parameter. The displacement of the Knudsen minimum reflects the departure from the slip model over a wider range of Knudsen numbers than predicted by theory.

Figure 3 illustrates the progressive departure of the measured inverse effective viscosity from the modified slip theory, and also plots the results of Jaffe[1] and Jensen *et al.*[2] for direct comparison. Generally, the departure from the slip model extends to a higher temperature than the calculations predict. This is particularly evident in the position of the Knudsen minimum in the inverse effective viscosity which is predicted to lie at a Knudsen number of $\simeq 0.5$ as compared to the experimental value of 0.75. The minimum region is illustrated in Fig. 4, showing the expected asymmetric behavior. The superfluid transition at 1.13 mK precludes following the data to lower Knudsen numbers without possible inaccuracies in deconvoluting the data in the superfluid.

A comparison to experiments performed in ^{4}He is instructive. The results of Whitworth[11] and Greywall[12] yield Knudsen minima for phonons at Knudsen numbers of 0.87 ± 0.13 and 0.65, respectively. The slip parameters measured in these experiments which used metallic and glass substrates are ~90% of those calculated by Jensen *et al.*

In conclusion, we have resolved the Knudsen minimum for a dilute gas of quasiparticles in ^{3}He.

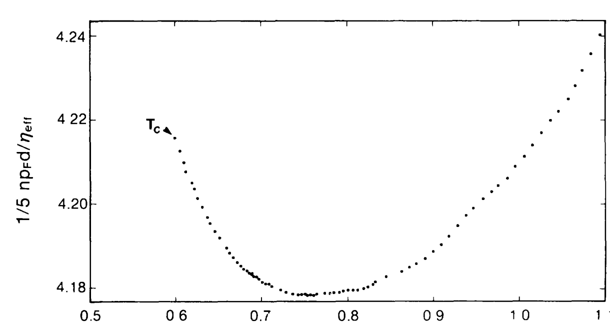

FIG. 4. The Knudsen minimum is illustrated in this figure. The minimum occurs at a Knudsen number of 0.75 as compared to 0.5 for the theory.

The results are in substantial agreement with experimentally determined slip parameters. Our results show that the theory systematically underestimates the departure of the effective viscosity from the slip model and overestimates the slip factor for our geometry. As a consequence, corrections to account for finite mean free paths in other experiments in the superfluid B phase would underestimate the true bulk viscosity and possibly obscure the expected shallow minimum in η/η_c at $\sim 0.4T/T_c$. Discrepancies with theoretical results indicate the need for further work with particular emphasis on the nature of the surface.

The authors would like to acknowledge the expert machining skills of Mr. R. McBrien and Mr. J. Schwede, and also the able assistance of Mr. J. Stinson and Mr. E. Yuen with the analysis of the data.

This work has been supported by the National Science Foundation, through Grants No. DMR-8018381 and No. DMR-8218279 and also the Research Corporation of America. One of us (J.M.P.) is the recipient of an Alfred P. Sloan Research Fellowship.

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