dict an ordering temperature of 28 ± 5 K. An equivalent calculation for a more compressed lattice with the same nearest-neighbor spacing as the dense-solid phase but with molecular axes allowed to tilt out of the film plane gave approximately the same ordering temperature,[9] the closer spacing being compensated by the formation of a rickrack configuration of the molecular axes. That a model which treats the surface film as an idealized, two-dimensional structure gives results in good agreement with the experimental observations is testimony to the essentially two-dimensional character of the transition.

Finally, we have to address the question of what are the ordered-state molecular configurations of the registered and dense-solid films. This would be a difficult problem even if we were dealing with a three-dimensional solid. If that were the case, we would expect to be able to deduce the ordered configuration from the intensity profile of the diffraction pattern,[10] assuming sufficient diffraction data could be collected. But there is too much background scattering from the substrate (particularly at larger scattering vectors) to make such an approach feasible with overlayer films. The only realistic alternative is to attempt to compare the observed intensities with those calculated assuming ordered configurations suggested by theory. We find, unfortunately, that either the two- or four-sublattice configuration can satisfactorily reproduce the observed diffraction profile of the registered-solid phase; the data available are not sufficient to allow us to choose between the two suggested alternatives. One other possibility remains, namely, to search for the extra superlattice reflections expected if the ordered state is the four-sublattice rather than two-sublattice configuration. As it happens, this too appears to be impractical; a calculation of the relevant structure factors makes it clear that none of the extra reflections could be detected with neutrons. Thus we are forced to conclude that, while we can study the onset and development of orientational order in the nitrogen overlayer, we cannot, at least at this time, determine the ordered configuration.

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Critical Velocities in Superfluid ^{3}He

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The oscillatory flow of superfluid ^{3}He through an 18-μm-diam orifice has been studied. A clearly defined critical velocity is seen in both the A and B phases. For velocities greater than the critical value, the superflow involves an excess dissipation. The temperature dependence of the critical velocity is reported for temperatures near the superfluid transition ($T/T_c > 0.9$).

The possibility of dissipationless flow is one of the most dramatic aspects of the superfluid state. The study of the limits of stable superflow and the mechanisms of critical velocities has proven a valuable test of our understanding of the nature of the superfluid state for both superconductors and liquid ^{4}He. We report in this Letter our first attempts to study the critical-velocity phenomenon in the new superfluid phases of liquid ^{3}He. For these experiments, we have chosen a geometry where the region of high velocity is restricted to a small well-defined region. In addition, we have used a toroidal flow region which allows, in the B phase at least, the application of the powerful condition of conservation of circulation.

The flow geometry used in this experiment consists of an annular channel blocked by a thin (10 μm thick) partition containing a single 18-μm-diam orifice. The partition and channel are constructed of Stycast 1266. A schematic view of the cell is shown in Fig. 1. The flow cell is mounted on a beryllium-copper torsion tube whose axis coincides with the axis of symmetry of the toroidal flow channel. The torsional resonant mode of the cell (near 1 kHz) is driven and detected electrostatically. This torsional motion produces an oscillatory superflow through the orifice.

The area of the orifice cross section is a factor of 800 smaller than that of the annulus. Consequently, the region of high-velocity flow is restricted to the immediate vicinity of the orifice. The design of this oscillator is similar to that employed in our earlier measurements[1] of the viscosity and superfluid density of ^{3}He.

In a typical measurement, the temperature is held nearly constant below the transition and the response of the superfluid to increasing amplitude of oscillation is observed. The voltage which provides the drive for the oscillator is slowly ramped up and the amplitude and period are recorded. For temperatures near the superfluid transition, the normal fluid is effectively locked to the oscillator through viscosity and only the superfluid is free to flow through the orifice.

In Fig. 2, data obtained in the B phase at a pressure of 20.8 bars and a reduced temperature of $T/T_c = 0.914$ are shown as a function of the amplitude, θ. For sufficiently small oscillation amplitudes, the drive voltage and amplitude are seen to be proportional. However, when the amplitude is increased beyond a critical value θ_c, the amplitude versus drive relation becomes nonlinear as an additional dissipation appears.

In Fig. 2, we also show the dependence of the period on the oscillation amplitude. It is seen that there is very little change in the period until the critical amplitude θ_c is reached. For amplitudes greater than θ_c, the period increases as the superfluid becomes coupled to the oscillator. Thus, we have two different signatures in the B phase for the detection of the critical velocity for superflow through the orifice.

In order to calculate the numerical values of the critical velocity from the critical amplitude θ_c, we have to consider details of the flow through

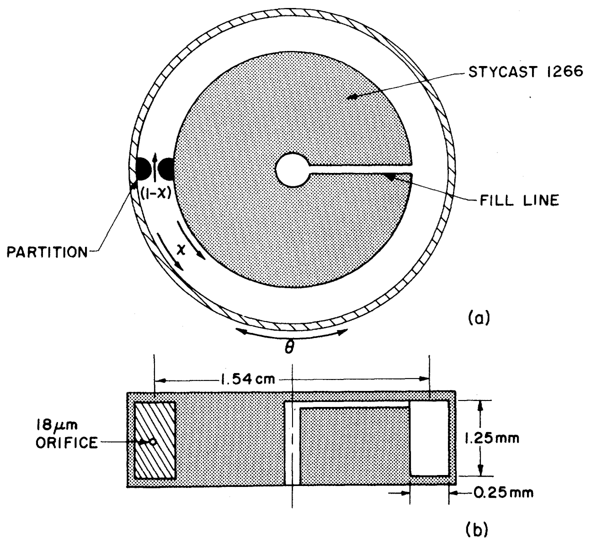

FIG. 1. Schematic of the experimental cell. (a) A view of the annular channel illustrating the induced flow, χ, and the backflow, $1-\chi$, through the orifice. (b) The cross section of the flow channel of diameter, $2R = 1.54$ cm, and area A, 0.25 mm by 1.25 mm.

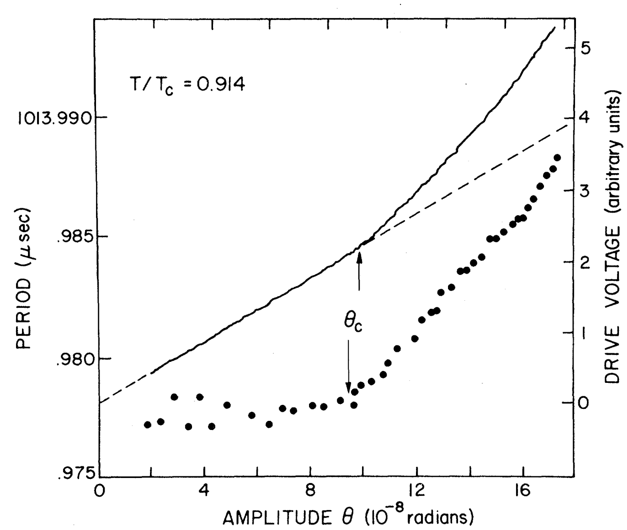

FIG. 2. B-phase results at 20.8 bars. The drive voltage (solid line), and period of the oscillator (closed circles) as the amplitude is increased are shown. The onset of additional dissipation is noted by the deviation of the drive voltage from a linear dependence (dashed line). The oscillator period at T_c is 1013.0503 μsec.

the orifice. It is important to note that even for a pure incompressible superfluid undergoing potential flow, a sizable fraction, χ, of the superfluid mass is directly coupled to the oscillator through the hydrodynamic impedance offered by the partition and orifice. For this case, the ratio of the fraction of fluid passing through the orifice to that coupled to the oscillator is given, to good approximation, by $(1-\chi)/\chi = 4\pi Rr/A$, where r is the radius of the orifice, R the mean radius of the annulus, and A the area of the annulus.[2] The fraction, χ, has been experimentally determined through calibration, using liquid ^{4}He at a temperature of 80 mK. The value, $\chi = 0.27$, obtained from this calibration, is in good agreement with a value obtained directly from the geometric parameters.

Assuming pure potential flow and neglecting shifts in ρ_s due to v_s^2 effects,[3] a mean superflow velocity $\bar{v}$ can be determined from the equation, $\bar{v} = 8\pi\chi f R^2\theta/a$, where f is the frequency of oscillation and θ is the angular excursion of the cell. Since θ has to be inferred from estimates of the spacing of the electrode structure and the stray capacitance of the leads, there is a systematic uncertainty in $\bar{v}$ which we estimate to be on the order of 25%.

The characteristics of the onset of excess dissipation in the superfluid can be illustrated by taking the ratio of the dissipation, Q^{-1}, to the dissipation at low amplitudes, Q_0^{-1}, and plotting this ratio against the average velocity in the aperture, $\bar{v}$. The result is shown in Fig. 3. The sharp onset of dissipation seen in this plot is reminiscent of that seen for the flow of superfluid ^{4}He through an orifice.[4,5] However, in the case of ^{4}He superflow much higher critical velocities, varying from 5 to 1000 cm/sec, are observed. We have repeated our measurements with superfluid ^{4}He. In this case, no superfluid dissipation is seen for velocities between 0.1 and 100 cm/sec, the maximum velocity attainable in our apparatus.

A series of measurements in the A phase at 28 bars have also been carried out. The data obtained are shown in Fig. 4. The drive and velocity ranges are similar to those for the B-phase measurements. Again, there is a low-velocity region where the dissipation is constant. As the drive voltage is increased, a break in the slope is seen at a critical value and, for higher velocities, the dissipation increases rapidly. However, in contrast to the B-phase measurements, the period increases continuously from the lowest amplitudes. At small amplitudes, the period shift may be due in part to reorientation effects of the l vector in response to the superfluid velocity field. In view of the complexities of the A phase, we are unable to offer, at this time, a completely

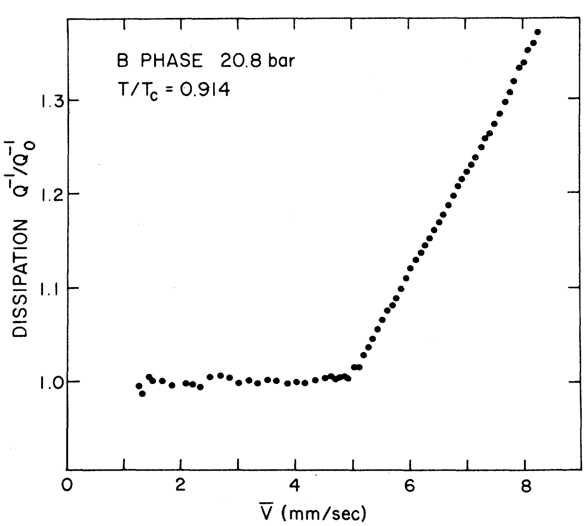

FIG. 3. A plot of the dissipation, Q^{-1}, normalized by the dissipation at small amplitudes, Q_0^{-1}, as a function of the average velocity, $\bar{v}$, in the orifice.

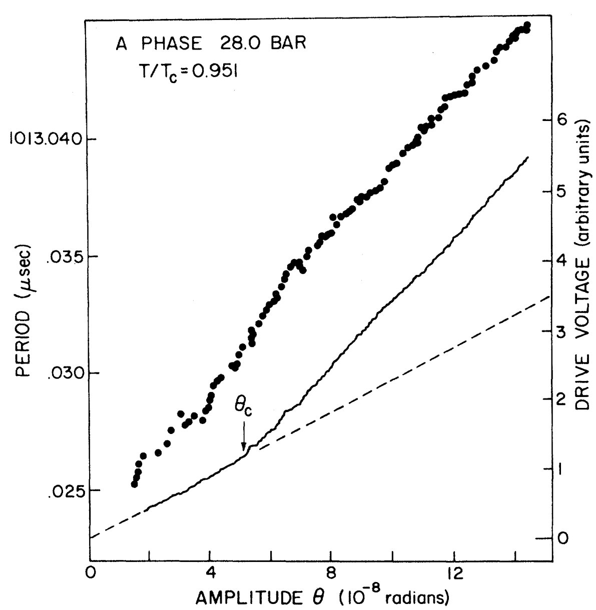

FIG. 4. The drive voltage (solid line), and period of the oscillator (closed circles) are shown for the A phase. The critical amplitude, θ_c, is seen only in the drive voltage data. The oscillator period at T_c is 1013.069 μsec.

satisfactory explanation of the A-phase period shift.

We have also measured the magnitudes and temperature dependence of the A- and B-phase critical velocities for a reduced-temperature range from $0.01 < t < 0.12$. These data are shown in Fig. 5. For the B phase, the correspondence between the onset of coupling of the superfluid mass and the appearance of dissipative superflow is excellent. The A-phase data shown pertain only to the onset of dissipative superflow.

A number of previous measurements in superfluid ^{3}He have demonstrated a variety of critical velocity phenomena. In contrast to the present measurement, where the region of high velocity is restricted to the immediate neighborhood of the orifice, these earlier experiments involve dissipative over rather large volumes. Johnson *et al.*[6] observed the heatflow of ^{3}He in 2- and 3-mm-diam tubes and have inferred critical velocities between 0.2 and 0.8 mm/sec in the A phase, whereas in the B phase they obtain values of 4 to 5 mm/sec. The temperature dependence is not discussed by these authors. Flint, Mueller, and Adams[7] have studied the NMR properties of flowing ^{3}He in 1.5-mm-diam tubes and 135-μm spaced parallel plates. Critical velocities of 0.64 to 0.12 mm/sec associated with the annihilation of the NMR signal are reported[7] in the A phase.

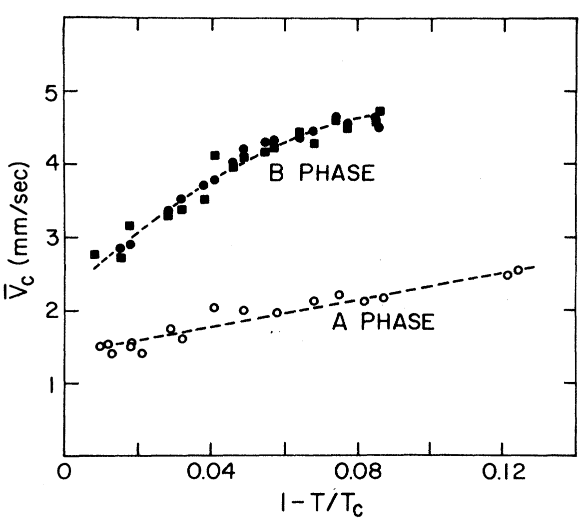

FIG. 5. The temperature dependence of the A- and B-phase critical velocities are shown. For the B phase, critical velocities obtained for the drive voltage (closed circles) are in good agreement with those obtained from the period (squares). In the A phase, the critical velocity is determined from the drive voltage (open circles). The dashed lines are shown as visual aids.

Recently, Bagley *et al.*[8] have reported measurements in the A phase using a torsional pendulum whose period is comparable to the orbital relaxation time of the textures. The flow geometry consisted of parallel plates separated by 49 μm. No region of dissipationless superflow was observed in this experiment. Rather, the dissipation was seen to increase from the smallest velocities, attaining a maximum at a superfluid velocity of 4 mm/sec. This dissipation was accompanied by a decrease in the period shift, implying that the entire superfluid was coupled to the pendulum. The effect, termed collapsed superflow, is thus associated with the texture in the superfluid. No such velocity dependence was observed in the B phase.

The comparison of our results to those of the existing literature is difficult since the geometries are very different. The majority of past experiments have dealt with the A phase, where the flow is affected by orbital relaxation, channel size, and flow geometry. The interpretation of the B-phase results is simpler. We have demonstrated that the superfluid does not couple to the oscillator in a dissipative manner for velocities below a clearly defined critical velocity. When this critical velocity is exceeded, the superfluid is increasingly coupled to the oscillator. This signature is seen to coincide with the onset of dissipative superflow. The critical velocity is of the order of several mm/sec and shows a temperature dependence which has not been observed by other workers. It is anticipated that this sensitive technique will be applied to other flow geometries to investigate the superflow more comprehensively.

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Rotational Speedups Accompanying Angular Deceleration of a Superfluid

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Exact calculations of the angular deceleration of superfluid vortex arrays show momentary speedups in the angular velocity caused by coherent, multiple vortex loss at the boundary. The existence and shape of the speedups depend on the vortex friction, the deceleration rate, and the pattern symmetry. The phenomenon resembles, in several ways, that observed in pulsars.

The angular deceleration ("spin-down") of a superfluid has gained much interest from the identification of pulsars as neutron stars and the well-founded prediction that the latter consist primarily of a neutron superfluid.[1,2] In particular, the abrupt changes in the rotation period, variously known as jumps, speedups, or anomalies ("glitches"), possibly originate in superfluid hydrodynamic processes.[3-6] Continuous vorticity models have not provided a mechanism for rotational anomalies although various vibration[7,8] and relaxation[9] phenomena have been studied. The new results of this Letter concern basic dynamical characteristics of discrete vortex arrays during unconstrained spin-down. These characteristics obviously invite comparison with those of pulsars, although only a relatively small number of vortices are used and only the grossest pulsar features are mirrored in the theoretical system.

I consider a rotating, two-component superfluid in which the vortices have singly-quantized circulation κ and are conserved except for possible annihilation at an exterior, circular boundary. To be consistent with the pulsar system, I assume the effective normal fluid is locked rigidly to the boundary (because of the strong magnetic field in neutron stars[1,2]). Also, the vortices are assumed to be unpinned at the boundary (a result of either the loss of $^3P_2$ superfluidity there or the existence of a boundary layer).[5] Thermodynamic processes and any difference between vortex dynamics in a sphere and a cylinder are disregarded for convenience.

The friction accompanying relative motion of the vortices and the normal fluid can be expressed as an angle θ relating the velocities of the kth vortex, v_k, of the local superfluid, $v_{s,k}$, and of the local normal fluid, $v_{n,k}$. In complex notation, appropriate for rectilinear vortices, this expression is[10]

$$v_k = e^{-i\theta}\left[v_{s,k}\cos\theta + iv_{n,k}\sin\theta + \frac{\nu'}{\rho_s\kappa - \nu'}(v_{s,k}-v_{n,k})\cos\theta\right], \tag{1}$$