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We have fabricated large ($\leq$400 μm diameter) high tensile stress SiN membrane mechanical resonators and measured the resonant frequency and quality factors (Q) of different modes of oscillation using optical interferometric detection. We observe that the dissipation (Q^{-1}) is limited by clamping loss for pure radial modes, but higher order azimuthal modes are limited by a mechanism which appears to be intrinsic to the material. The observed dissipation is strongly dependent on size of the membrane and mode type. Appropriate choice of size and operating mode allows the selection of optimum resonator designs for applications in mass sensing and optomechanical experiments. © *2011 American Institute of Physics*. [doi:10.1063/1.3671150]

Nanomechanical resonators are well-suited to use as sensors, oscillators, and filters.[1] The high resonant frequencies of these devices[2] together with their low dissipation and mass are also useful for fundamental investigations of the boundary between classical and quantum mechanics. In this regard, stoichiometric high stress (>1 GPa) SiN resonators have been the target of wide interest due to their extremely high quality factors (Q).[3–6] Recently, quality factors of over a million have been observed on fully clamped silicon nitride beams[6–8] and membranes.[4,5,9,10] These thin silicon nitride membranes are also used in high finesse optical cavities where their low mass and high mechanical quality factors prove ideal for optomechanical experiments.[3–5]

High tensile stress silicon nitride membranes have Qs two orders of magnitude higher than stress free silicon nitride and other amorphous materials at low temperature.[10] This is a departure from the universality observed in glasses and has been primarily attributed to the enhancement of the stored energy relative to the losses induced in the high tensile stress oscillator.[8] Recently, it has been reported that the Q of these membrane resonators strongly depends on the mode shapes due to the nature of the clamping losses associated with the different modes.[9,11] In this article, we report the limits of observed dissipation (Q^{-1}) of circular membrane resonators and its dependence on frequency, mode type, and diameter. For large circular membranes resonating with no radial nodes (cake-like-modes), the dissipation falls off as the number of azimuthal nodes increases. However, at some point, the resonators' Q seems to be limited by a dissipation floor whose origin is other than clamping loss; further addition of cake like nodes result in an increased dissipation. This dissipation floor seems to be both frequency and size dependent. At higher frequencies, the modal dependency of dissipation vanishes and the dissipation seems to be dominated by the dissipation floor. These results provide insights into the optimum resonator designs for achieving high Qs.

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Stoichiometric tensile (~1.0 GPa) silicon nitride films (110 nm-thick) are grown on top of 660 nm of thermal oxide on a silicon substrate. We fabricated optically defined silicon nitride circular resonators (50 μm to 400 μm in diameter) with etch release holes (1 μm diameter) that are 5 μm apart on silicon oxide/silicon. Releasing the resonators in buffered hydrofluoric acid (6:1) resulted in the honeycomb structure visible in the resonator back plane while producing a device with an average thickness of 27 nm (Figure 1). Thicknesses were measured after peeling off the resonator from the supporting nitride using polydimethylsiloxane (PDMS) as an adhesive supporting layer. Subsequently, we image the back plane with atomic force microscopy (AFM), as shown in Figure 1(b). Resonance frequencies and Q factors of the resonators are measured using interferometric means[12] with a piezo actuation under vacuum (3×10^{-7} Torr).

The resonant frequency $\omega_{n,m} = 2\pi f_{n,m}$ of the taut membranes with tensile stress (σ) is given by[13]

$$\omega_{n,m} = 2\xi_{n,m}\frac{c_R}{D}. \qquad (1)$$

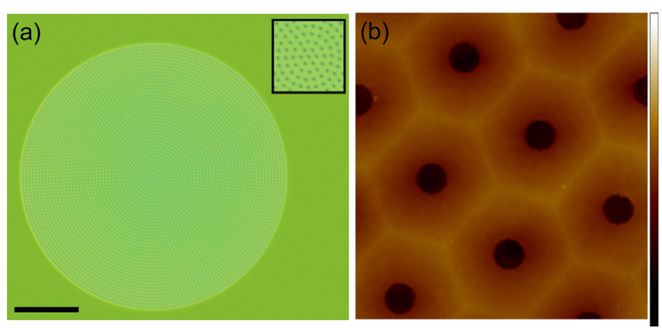

FIG. 1. (Color online) (a) Optical micrograph of a 400 μm diameter high stress silicon nitride membrane. Inset is a zoomed-in optical image of the central region illustrating radial symmetry of the circular perforations. Scale bar corresponds to 100 μm. (b) AFM image (14.1 μm × 14.1 μm, color scale, 0 to 100 nm) of the backside of the released membrane showing a honeycomb structure (average thickness 27 nm) resulting from the isotropic etching of the membranes via circular etch holes. Released membranes were peeled off using a PDMS support layer and subsequently AFM imaged to determine, both, uniformity and thickness of the nitride membranes.

Modes of these resonators are indicated by indices (n, m). The mode number n = 0, 1, 2,… corresponds to azimuthal nodes and m = 1, 2, 3,… indicates the number of radial nodes with n = 0, m = 1 being the fundamental mode which has no azimuthal node, hence it a pure radial mode. Here D is the diameter of the resonator, $c_R = \sqrt{\sigma/\rho}$ is the phase velocity. $\xi_{n,m}$ is the mth zero of the Bessel function J_n (x). The resonant frequencies and dissipation of different modes are shown in Figure 2 for a 400 μm resonator. Resonant frequencies of the different modes of the membranes follows Eq. (1), clearly indicating the stress dominated resonance characteristics (phase velocity ~540 m/s). Circularly symmetric etch release holes (Figure 1(a), inset) result in predictable mode positions in frequency space. Dissipation among individual "family" members (n = 0, 1, 2, 3,…, m = constant, i.e., azimuthal harmonics for low radial mode numbers, m ≤ 2) of large resonators consistently show a reduction in dissipation which then reaches a dissipation floor for the higher azimuthal harmonics (Figure 2). Similarly, when we examine the Q^{-1} of higher order pure radial modes (m = 3, 4,…) and their azimuthal harmonics (n = 0, 1, 2,…), these modes also show a drop in dissipation; however, their mode dependence gets progressively weaker, as shown in Figure 2.

We examined the Q^{-1} of resonators of various diameters to explore whether this dissipation is frequency or mode dependent. As expected from Eq. (1), the resonant frequencies of the respective fundamental modes (n = 0, m = 1) show an inverse relationship with diameter. The dissipation of the observed fundamental modes (n = 0, m = 1) of resonators of different diameters are shown in Figure 3 which seem to be independent of the diameter. In the same figure, we plot modes with n = 0, 1, 2,…, m = 1 (as in Fig. 2) connected by thin solid lines. The observed dissipation of several resonators appears to suggest a dissipation floor which depends on the size and frequency of the resonator with fQ and $Q^{-1}D$ = constant. This relationship (fQ = constant) is plotted in the figure as a solid straight line (fQ = 10 THz). The frequency dependence of the dissipation floor is more pronounced in the smaller resonators with high resonant

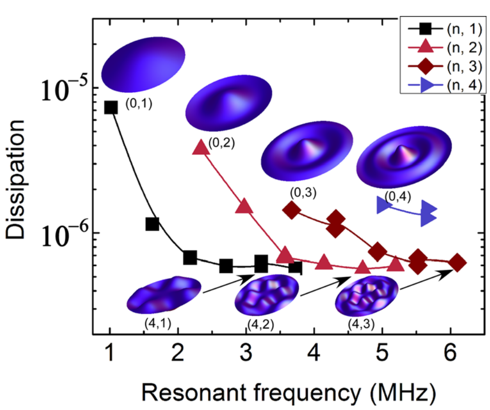

FIG. 2. (Color online) Modal dependence of dissipation for optically defined membranes (400 μm, 27 nm average thickness) showing a reduction in the dissipation with increasing azimuthal harmonics (n = 0, 1, 2,…, for a given m ≤ 2). For modes with higher azimuthal harmonic numbers, dissipation gradually starts to increase indicating a frequency dependent dissipation floor. Lines (splined fits) are drawn between the points to guide the eye to follow the dissipation trend within an individual family of modes.

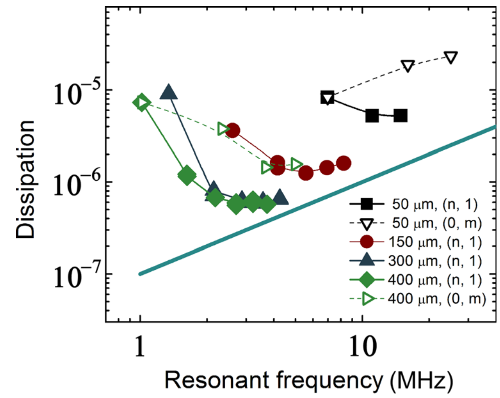

FIG. 3. (Color online) Dissipation and resonant frequency as a function of diameter and vibrational modes of the resonator. Filled data points connected by solid lines denote the pure radial mode along with its azimuthal harmonics (n = 0, 1, 2, 3,…, m = 1). Open triangles connected by dashed lines show the series starting from the fundamental mode and purely radial harmonics (n = 0, m = 1, 2, 3), i.e., no azimuthal harmonics. Resonators show dissipation dominated by clamping losses for the fundamental mode and low order azimuthal modes whereas higher azimuthal harmonics more closely follow the dissipation floor. The solid line delineates a dissipation floor corresponding to a fQ product of 10 THz.

frequencies where the dissipation is nearly mode independent. Also seen in Figure 3 is the ascending radial harmonics (n = 0, m ≥ 1, connected by dashed lines) for resonators of two different sizes. Pure radial modes for large resonators (D > 200 μm) show a reduction in dissipation as the number of radial nodes increases (n = 0, m = 1, 2, 3,…). However, smaller resonators (D < 100 μm) show an *increase* in dissipation with higher radial mode numbers (n = 0, m = 1, 2, 3,…).

Dissipation in high stress resonators has been discussed in earlier articles.[6,8–10,14] In general, the observed dissipation of these taut membranes and strings is strongly dependent on the size and mode numbers of the resonator.[6,8,9,14] Clamping losses are identified as a significant fraction of the overall dissipation for pure radial modes (n = 0) in taut membranes.[9] In general, the clamping loss contribution is predicted to have following dependence:

$$\frac{1}{Q} \propto \frac{t}{D}. \tag{2}$$

Here, t is the thickness of the resonator and the prefactor has a strong mode dependence. For higher order azimuthal modes of the high stress drums, an exponential drop in dissipation within the individual radial mode "family ($\omega_{n=0,1,2,\ldots,m}$)" is predicted. This can be understood in terms of destructive interference between the acoustic waves radiated by different equivalent segments of the resonator's periphery.[9] This reduction in dissipation by an order of magnitude within an individual family has been observed for large resonators. However, the observed behavior of the dissipation in our nitride membranes is not entirely consistent with current clamping loss models. In Figure 3, we note the following discrepancies: (1) All resonators' dissipation should decrease as the diameter increases (Eq. (2)). For large resonators, the dissipation of the fundamental mode is nearly constant. (2) Consider modes where n ≥ 0, m = 1. With increasing azimuthal mode number, the largest resonators show a decreased Q^{-1} with larger n. However, in the smaller resonators, Q^{-1} only decreases for $n = 0$, $n = 1$. Modes with $n \geq 2$ show a constant or increasing Q^{-1} (e.g., 150 μm devices). (3) If we examine the pure radial modes of the largest and one of the smallest resonators ($n = 0$, $m = 1, 2, 3, \ldots$), the largest resonator shows a decrease in dissipation with increased radial node number, while the smaller resonator shows an increased dissipation for the same mode variation (Figure 3, triangles). We conclude that resonators reach a dissipation floor for higher azimuthal harmonic numbers. This dissipation floor is strongly dependent on resonator diameter. The aggregate behavior where frequency/Q^{-1} appears to reach a limiting value argues that clamping loss alone cannot be responsible for the observed behavior.

Recently, Unterreithmeier *et al.*[8] developed a model for the influence of pre-stress on the internal dissipation of silicon nitride strings. They observed that the quality factor of these essentially one-dimensional resonators decreased for higher harmonics. This has been attributed to the enhancement of local strain in the resonator associated with the increased local curvature as mode number increases. The fundamental mode of shorter strings had much lower quality factor compared to the higher harmonic of the larger strings of similar frequencies, attributed to the additional curvature at the clamping points of the smaller resonators. It is possible that the observed behavior in our resonators results from a combination of clamping losses and internal losses. Clamping losses appear to be dominant for the observed fundamental modes of large resonators, but for higher azimuthal harmonics, the observed floor and subsequent increase in dissipation could be due to the internal losses (which is modified by pre-stress, in a manner similar to the strings[8]). Recent investigation on low stress silicon nitride membranes has shown that quality factor can be strongly frequency dependent due to the mode-coupling between the membrane modes and matching resonances of partly suspended support.[15] However for the high stress nitride membranes reported in this article, the support does not present well-defined resonances in the relevant frequency range given the mounting of the chip containing the membrane resonator which is fully attached on its whole back plane onto the piezoelectric oscillator. We can surmise that a finite fQ limit (10 THz) arises due to some intrinsic dissipation mechanism in the amorphous material.

In conclusion, we have measured the dissipation in high stress silicon nitride membranes as a function of frequency, size, and mode shape of the resonator. The observed Q's of the resonators are strongly mode dependent for large drums ($D > 200\,\mu$m) where dissipation decreases for increasing aziumthal harmonic numbers (for low radial harmonic numbers, $m \leq 2$). However, higher frequency modes of larger resonators ($D > 200\,\mu$m) and all the modes of smaller drums ($D < 200\,\mu$m) seem to be significantly dominated by a dissipation floor which reduces the modal dependence of the dissipation. We measured high Q factors ($\approx$2 million) for ultrathin membranes (average thickness 27 nm) with high surface to volume ratio. This offers unique potential for surface activated sensors and optomechanical experiments. Silicon nitride resonators are actively pursued for use in optomechanical experiments with the possibility of using ultra-thin nitride membrane resonators in high finesse cavities,[3] where high mechanical Q, low mass, and low spring constants are desirable. In this regard, large ultrathin resonators that can exhibit high quality factors as fabricated here should provide a useful means to achieve and explore the interaction between optical and mechanical fields.

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