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Transport in Mesoscopic ^{3}He Films on Rough Surfaces

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Abstract Measurements of the flow of thick ^{3}He films over a highly polished silver surface, using a high precision torsional oscillator, have found unexpectedly long momentum relaxation times[1]. This results in a decoupling of the normal state helium film from the oscillator motion at low temperatures. In the ballistic regime the relaxation rate varies linearly with temperature. This result is explained by a theory in which the effect of surface roughness is incorporated into the transport equation as a disorder potential[2]. This potential is fully characterised experimentally by AFM measurements of the surface roughness power spectrum, which we find to have a stretched exponential form. Incorporating the measured spectrum into the theory allows a direct comparison with the observed relaxation rate, with good agreement. The anomalous temperature dependence of the relaxation rate is accounted for by interference between bulk inelastic scattering and this weak surface elastic scattering. Decorating the substrate with scattering centres restores film-substrate coupling at the measurement frequency of 2.5 kHz. These results on the nature of quasiparticle scattering at rough surfaces may have implications in the understanding of the effects of confinement on superfluid ^{3}He.

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1 Introduction

The effect of a wall on mesoscopic systems is of interest, especially in the light of recent technological advances that have led to many ballistic systems where the mean free path of particles exceeds the system size. Boundary scattering phenomena are extremely important in understanding the transport processes in such systems. The nature of the elastic scattering

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processes at the boundary has to be included in any microscopic theory that attempts to explain the effect of the boundary on transport in such systems. In certain cases, the boundary effect can interfere with bulk scattering processes leading to interesting novel physics.

^{3}He is a quantum fluid that is well-described by Landau's Fermi liquid theory[5] at temperatures below 100mK down to the critical temperature < 2mK where it undergoes a transition to the superfluid state. The relevant excitations in Landau's Fermi liquid theory are quasiparticles that move ballistically at the Fermi velocity, v_F with momentum $p_F = m^\star v_F$ where $m^\star$ is the quasiparticle effective mass. The Fermi energy $E_F \sim 1$K and the typical Fermi wavelength $k_F^{-1} \sim 1.7\AA$. In the normal state of ^{3}He, the inelastic mean free-path for quasiparticles scattering off one another is $\lambda_{in} \propto T^{-2}$. At low temperatures, this inelastic mean free path can be as large as 65μm at 1mK. Also, the viscous penetration depth of ^{3}He, $\delta = \sqrt{2\eta/\omega\rho} \sim 5 \times 10^5/T$ (nm mK) where η is the viscosity, ω is the frequency and ρ is the density of the fluid. At low temperatures, $\delta \sim 500\mu$m at 1mK. For ^{3}He confined in a thin film of thickness, $L < \lambda_{in}$, and $L \ll \delta$ we have a ^{3}He film that can be treated as a surface boundary layer. Slip corrections in this regime have been studied and reviewed by Einzel and Parpia[3]. More recently, intriguing results were produced by torsional oscillator experiments[1,4] done by Casey *et. al*[1] where ^{3}He films were in contact with highly polished silver surfaces, which were characterised by AFM surface scans. A microscopic theory of surface scattering by Meyerovich[2] appears to explain the basic observations. An application of this theory to include the exact structure of the surface as measured by the AFM scan technique is the subject of this paper.

2 Review of Experiment

In the experiments by Casey *et. al.*[1], a ^{3}He film of thickness L in the range of 100 to 300nm is condensed onto a highly polished silver substrate. This substrate is located at the head of a torsional oscillator operated at 2841 Hz. The ^{3}He film thickness is much smaller than the viscous penetration depth and the film can be treated as a surface boundary layer. The frequency shift of the torsional oscillator and the dissipation due to the film were measured[1]. At high temperatures (above 100mK), the frequency shift is independent of temperature indicating full mass loading. As the temperature is lowered, the frequency shift is observed to reduce indicating that the motion of the film decouples from that of the substrate. This is accompanied by an increase in the dissipation of the film. This data is used to deduce the temperature dependence of $\omega\tau$, where τ is the momentum relaxation time of the film in the torsional oscillator and ω is the oscillator frequency. The temperature dependence is reported to be linear between 10mK and 100mK, and also seems to be independent of the thickness of the film. Decorating the substrate with large silver particles and hence increasing its roughness is observed to kill this effect. This suggests that the decoupling arises from the effect of surface scattering off the polished silver substrate. The reported temperature dependence can be explained by a theory for surface scattering by Meyerovich[2].

3 Review of Theory

Meyerovich and Stepaniants[2] consider a model of multiple parallel surfaces in a mesoscopic geometry between which a liquid is confined. In our case, we have a ^{3}He film and hence one surface is the wall and the other surface is free. The authors map the Hamiltonian of a free particle and a random rough wall to an equivalent problem with flat walls and a random bulk distortion. They then obtain an expression for the wall-induced transition probability, $W(\mathbf{q},\mathbf{q}')$ in terms of a correlation function, $\zeta(y,z)$ that characterizes the surface roughness profile.

$$W(\mathbf{q},\mathbf{q}') = \frac{\pi^4}{m^{\star 2}L^6}\,\zeta(y,z)\,(\frac{p_x L}{\pi\hbar})^2\,(\frac{p'_x L}{\pi\hbar})^2 \tag{1}$$

where x is the direction perpendicular to the suface and $\mathbf{q}$ and $\mathbf{q}'$ are quasimomenta in the plane of the film. The effective relaxation time is given by the expression

$$\frac{1}{\tau_{eff}(\mathbf{p})} = \frac{1}{\tau_b}\Big(1+\int \frac{W(\mathbf{p},\mathbf{p}')}{(\varepsilon(\mathbf{p}')-\mu)^2/\hbar^2+1/4\tau_b^2}\,\frac{d\mathbf{p}'}{(2\pi\hbar)^3}\Big) \tag{2}$$

where $\tau_b = \lambda_{in}/v_F$ is the inelastic relaxation time of bulk ^{3}He and $\varepsilon(\mathbf{p})$ are the quasiparticle energies for partices of momenta $\mathbf{p}$. For instance, if the surface roughness power spectrum is of Gaussian form,

$$\zeta(\mathbf{s}) = \ell^2 exp(-s^2/2R^2) \quad ; \quad \zeta(\mathbf{q}) = 2\pi\ell^2R^2 exp(-q^2R^2/2\hbar^2) \tag{3}$$

with R being the average distance between inhomogeneities of height ℓ. Meyerovich[6] derived the expression for the relaxation time for this case. and finds that τ_{eff} has a linear temperature dependence. Bowley and Benedict[7] apply this theory, for the case of Gaussian surface correlations in the geometry of the torsional oscillator, to derive the measured relaxation time in the oscillator, τ_{osc} as measured by Casey *et. al.*[1] and find that $\tau_{osc}^{-1} \propto T$.

4 Calculation

AFM scans of the silver substrate used in the experiments[1] find a stretched exponential form for the surface correlations. We have applied Meyerovich's theory with this form for the roughness power spectrum and calculated the relaxation time in the torsional oscillator. We derive the expression for the effective relaxation time τ_{eff} along the lines of the derivation by Meyerovich to obtain

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_b}\Big(1+\frac{k_F^2}{2\sqrt{2}\pi^2}\sqrt{k_F\lambda_{in}}\,(\frac{k_F}{L})\int_0^{\pi} d\theta\, cos^2\theta \int_0^{2\pi} d\alpha\,\zeta(\theta,\alpha)\Big) \tag{4}$$

where $p_x = \mid \mathbf{p} \mid cos\theta$ and α is the angle between $\mathbf{q}$ and $\mathbf{q}'$. We make a change of variables from (θ,α) to (u,v) such that $\zeta(u,v)$ is the Fourier transform of the surface height profile measured by AFM. We derive an expression for the relaxation time measured in the torsional oscillator, τ_{osc} in the following form

$$\frac{1}{\tau_{osc}} = \frac{3\sqrt{2}}{32\pi}\,v_F\,(\frac{k_F}{L})\sqrt{k_F\lambda_{in}}\int\int_{S'} \mid \zeta(u,v)\mid^2\,\frac{w}{s}\sqrt{1-w^2}(w^2-2ws+1) \tag{5}$$

so τ_{osc} may be computed using the (Fourier transform of) AFM data directly. Here w and s are functions of u and v.

$$w = \frac{u}{k_F}+\sqrt{1-\frac{v^2}{k_F^2}} \quad ; \quad s = \sqrt{1-\frac{v^2}{k_F^2}} \tag{6}$$

For the oscillator operated at 2841 Hz, the results for $\omega\tau_{osc}$ as calculated using the above expressions is shown in Fig.1. The linear temperature dependence of the relaxation time is recovered by the theory with one fitting parameter, viz., the thickness of the film, L. The fit value for L is within the range of film thickness used in the experiment. The calculated expressions, equations 4 and 5, reduce to the expressions for τ_{eff} and τ_{osc} derived by Meyerovich and Bowley *et. al* respectively, in the limit of Gaussian surface correlations.

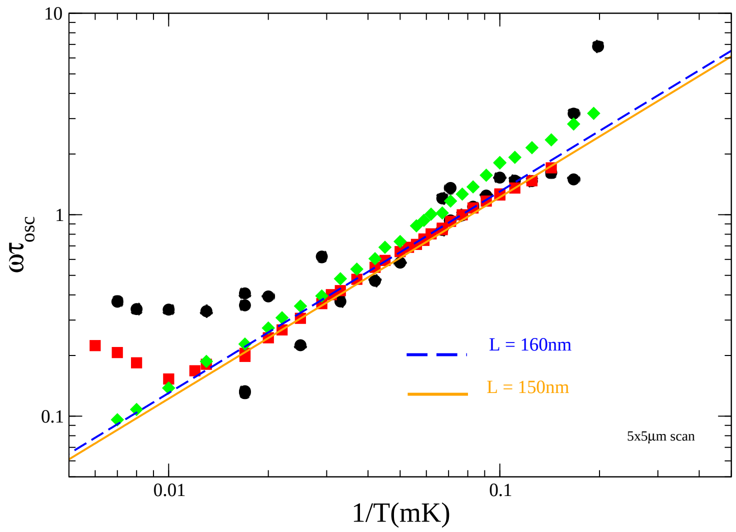

Fig. 1 (Color online) Temperature dependence of ^{3}He slab-substrate relaxation time in the torsional oscillator, $\omega\tau_{osc}$. Lines are theoretical calculations using AFM scan data for the roughness power spectrum. The data points are experimental data taken by Casey *et. al.*[1] for films of nominal thickness 100nm(circles), 240nm(diamonds) and 350 nm(squares).

5 Conclusions

The calculated relaxation times agree reasonably well with those measured experimentally, in terms of the thickness of the film which is used as a fitting parameter. The linear temperature dependence of the relaxation rate is recovered. However, the dependence of the relaxation rate on the thickness of the film as predicted by the theory is not observed experimentally and is a matter for further investigation. The theory explains the temperature dependence as arising from an interference between elastic wall scattering and bulk ^{3}He quasiparticle scattering. The bulk mean free path, λ_{in} is comparable to the thickness of the film in this temperature range and in a simple picture, quasiparticles scattering off the wall return to scatter from the wall after one or more bulk scattering events. This may be understood as the *classical*[6] interference of bulk and wall scattering processes that gives rise to this anomalous temperature dependence. For a surface with a Gaussian roughness profile, the transverse momentum change produced by surface scattering is $\hbar/R$ where R is the surface correlation length as defined in equation 3. This understanding is expected to be useful in experiments on thin slabs of superfluid ^{3}He in confined geometries. Here quasiparticles scattering off the surface determine the suppression of the order parameter at the surface and the density of states in the gap. This work generalizes the theory of Meyerovich and Stepaniants[2] to a surface with an arbitrary roughness power spectrum. This is a quantity that is readily accessibly by experiment using modern surface probe techniques. The intriguing behaviour of confined ^{3}He on rough surfaces also realizes a mesoscopic system in which the disorder potential is fully characterised and measurable.

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