

# Logarithmic Temperature Dependence of the Sound Speed in Amorphous Silica at Low Temperatures

A. Fefferman · R.O. Pohl · J.M. Parpia

Published online: 6 June 2007  
© Springer Science+Business Media, LLC 2007

**Abstract** The tunneling model has enjoyed considerable success in describing the low temperature properties of glasses. However, departures from the tunneling model have been noted in experiments at very low temperatures (below 100 mK). We have measured the change in the sound speed  $\Delta V/V_0$  between 1 and 40 mK in an amorphous silica double paddle oscillator oscillating at 14 kHz. Most importantly, the sound speed displayed the logarithmic temperature dependence predicted by the tunneling model to a lower temperature than in previous experiments on amorphous silica. Below 3 mK the sound speed departed from the logarithmic temperature dependence and began to level off. The leveling off can be explained by either an intrinsic effect or thermal decoupling of the sample from the thermometer. By heating the oscillator with a gamma source to determine the thermal resistance of the oscillator, it was found that a relatively large stray heat input ( $9 \times 10^{-4}$  nW) would be needed to cause the leveling off. Further work will be needed to determine whether the leveling off is due to stray heat or intrinsic physics.

**PACS** 61.43.Fs · 62.65.+k

## 1 Introduction

The tunneling model [1–3] has been proposed to explain the universal low temperature properties of glass. According to the tunneling model, some of the atoms or groups of atoms in a glass can tunnel between nearly degenerate potential energy minima, forming two level systems (TLS). The distribution of asymmetries and tunneling barriers of the double well potentials in these TLS's is such that the probability of a TLS having a particular energy splitting  $E$  is approximately independent of  $E$ . In

A. Fefferman (✉) · R.O. Pohl · J.M. Parpia  
Department of Physics, Cornell University, Ithaca, NY 14853, USA  
e-mail: adf24@cornell.edu

its simplest form, the tunneling model assumes that the TLS do not interact with each other. Under this assumption, the dependence of the sound speed on temperature is given by [4]

$$\frac{\Delta V}{V_0} = C \ln \frac{T}{T_0} \quad (1)$$

where  $V_0$  is the speed of sound at some reference temperature  $T_0$  and  $C$  is the tunneling strength. The sound speed has an additional term that predicts low temperature saturation in the sound speed, but this term is negligible, as in the present experiment, for  $\hbar\omega \ll k_B T$ , where  $\omega$  is the phonon angular frequency.

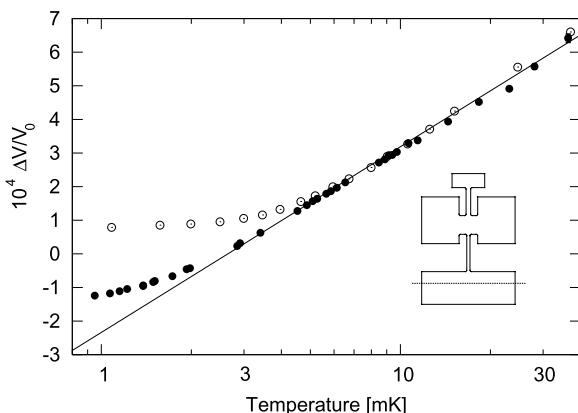
Several recent experiments have supported theories that incorporate interactions between the TLS. An overview of such experiments is given by [5] and references therein. The dipole gap model of TLS interactions was described in [7], and the acoustic measurements in [6] were interpreted in terms of TLS interactions. Agreement between measurements of the sound speed in silica and (1), which does not take into account interactions, has been mixed [8–12]. In [8, 12], the behavior of the sound speed depended on the strain level, suggesting non-linear and heating effects. In [10], a departure from the logarithmic temperature dependence was observed at 25 mK. In [9], leveling off was not observed, and the measurements were extended down to 6 mK. In [11], sound speed measurements were extended to 1 mK, and a departure from the logarithmic temperature dependence was observed at 6 mK. An extension of the range of logarithmic temperature dependence to ever lower temperatures has therefore been observed as experimental techniques have been refined.

When conducting low temperature experiments on dielectric glasses, it is important to be careful of temperature gradients between the sample and the thermometer due to heat input from a variety of sources. The saturation of the sound speed in [10] was initially attributed to intrinsic physics, namely a low energy cut off in the tunneling density of states  $\Delta_{0,\min}$ . However, it was later discovered [11] that the saturation could also be attributed to thermal decoupling of the sample from the thermometer. Thermal relaxation of tunneling states, cosmic rays, radiation from building material, vibrations, and high strain are all potentially significant sources of heat in low temperature experiments on dielectric glass. An analysis of the heat input to the sample from these sources is given in [11]; the primary contribution to thermal decoupling in that case was relaxation of the tunneling states, with a lesser contribution from cosmic rays. In the present experiment, we used a different type of oscillator, as described below, as well as additional vibration isolation to reveal that the range over which the sound speed is logarithmic extends down to at least 3 mK. We have been unable to find a unique explanation for the weaker temperature dependence observed below 3 mK: it could be due to intrinsic physics or thermal decoupling. Additional innovations will be required to determine the intrinsic temperature dependence of the sound speed below 3 mK.

## 2 Experiment

A double paddle oscillator nearly identical to the small oscillator described in [9] was used to measure the sound speed in the present experiment. The double paddle

**Fig. 1** Speed of sound versus experimental plate temperature on a semi-logarithmic plot. *Solid circles*: present work; *open circles*: taken from Fig. 4 of [11]. The *solid line* represents the logarithmic dependence predicted by (1). Also shown is the geometry of the oscillator used for the present work, with the thin *dotted line* marking the clamping position (from [9])



oscillator with the geometry shown in the inset of Fig. 1 was laser cut from a 0.4 mm thick plate of vitreous silica. Such double paddle oscillators can be operated in a variety of modes [13]. The 14 kHz mode used in the present experiment is one of the antisymmetric torsional modes. The oscillator was entirely coated with a 1 micron thick silver film to aid in thermalization. The silver film was also used for electrostatic excitation and detection of the oscillator. In the present experiment, a LabView program was used to sweep the oscillator excitation frequency through the resonance slowly enough so that no time dependent effects were observed. The response of the oscillator was measured with a lock-in amplifier, and the resonant frequency (proportional to the sound speed) was defined as the frequency at maximum amplitude. Care was taken to drive the oscillator at a low enough level so that non-linear and heating effects were negligible. A negligible shift in the resonant frequency was observed when the strain level was increased by a factor of two, even at 3.6 mK.

A dilution refrigerator with a PrNi<sub>5</sub> nuclear stage was used to cool the sample. Airlegs were used for external vibration isolation, and additional vibration isolation was provided by a copper mass-spring system used to suspend the sample over the experimental plate. A <sup>3</sup>He melting curve thermometer calibrated to the solid and superfluid A fixed points was used to measure the temperature of the experimental plate. A more detailed description of the cryostat can be found elsewhere [14].

Figure 1 displays the sound speed measured with the double paddle oscillator at 14 kHz between 1 mK and 40 mK (solid circles). Also displayed in Fig. 1 (open circles) is the sound speed measured with a cylindrical torsional composite quartz-silica oscillator at 84 kHz between 1 mK and 40 mK as reported in [11]. Both experiments show a logarithmic temperature dependence of the sound speed above 6 mK, as predicted by (1). Below 6 mK, the sound speed in the composite oscillator shows an increasingly weak temperature dependence as temperature decreases. In contrast, the sound speed in the double paddle oscillator begins to level off from the logarithmic temperature dependence only below 3 mK.

The true logarithmic dependence of the sound speed between 6 mK and 3 mK that was masked by thermal decoupling in [11] was revealed in the present experiment because of the more than 100 times lower thermal resistance of the sample in

the present experiment. To a lesser extent, the smaller mass (and therefore decreased internal heat release) of the double paddle oscillator compared to the composite oscillator also contributed to a smaller temperature gradient between the sample and the thermometer. Since driving the oscillator into the nonlinear regime causes a decrease in the resonant frequency [15], nonlinear effects could counteract an apparent increase in the resonant frequency due to thermal decoupling. However, we were careful to drive the oscillator at a low enough strain so that any non-linearity had a negligible effect on the sound speed (as discussed above).

The leveling off observed in the present experiment below 3 mK may be due either to intrinsic physics or thermal decoupling. In order to determine the stray heat input needed to produce the leveling off, the thermal resistance between the sample and the experimental plate was measured by introducing a known heat input  $\dot{Q}$  to the sample and measuring the resultant shift in sound speed. The heat input was provided by gamma radiation from a cesium source situated outside the cryostat. The radiation significantly heated the sample (as evidenced by the shift in sound speed upon heating) but did not alter the temperature of the experimental plate (as evidenced by a negligible shift in the melting curve thermometer reading). The measurement of sound speed versus temperature that was made before the cesium source was introduced was used to relate the sound speed shift upon gamma-induced heating to a temperature shift  $\Delta T$ , yielding a thermal resistance  $R_{\text{th}}(T) = \Delta T / \dot{Q}$ . We conclude that a stray heat input of  $9 \times 10^{-4}$  nW would be required to produce the leveling off observed in the present work.

The saturation observed in [11] was modeled by a stray heat input of only  $2 \times 10^{-4}$  nW/g, and only a portion of this stray heat input was due to thermal relaxation. Therefore, under the assumption that the amount of thermal relaxation is not sample dependent, we conclude that the stray heat input due to thermal relaxation cannot account for the entire amount of heat necessary to produce the leveling off in the 0.1 g double paddle oscillator. No strain heating was observed in the present experiment, and heating due to radiation from building materials and cosmic rays was negligible compared to the required heat input. Vibrational heating apparently was significant: using a copper mass-spring system to suspend the oscillator over the experimental plate caused a decrease in the heat input needed to explain the leveling off from  $15 \times 10^{-4}$  nW to  $9 \times 10^{-4}$  nW. It is possible that even with the mass-spring filter in use, vibrational heating amounts to the remaining  $9 \times 10^{-4}$  nW of stray heat input in the present experiment.

### 3 Conclusion

The leveling off of the sound speed observed below 3 mK in the present experiment may be due to stray heat input or intrinsic physics. It is unclear what the mechanism for an intrinsic effect might be. A low temperature saturation is expected [16] in glasses composed of atoms with nuclear quadrupole moments, but the constituent atoms of silica do not have nuclear quadrupole moments. If the leveling off is due to an intrinsic effect, and if a subset of the TLS contributing to acoustic properties of the glass also contribute to its dielectric properties, one would expect to observe a leveling off of the dielectric constant at low temperatures as well. Saturation in the

dielectric constant of  $\text{SiO}_x$  at low temperatures has been observed [17], but, as in the present work, heating from an external source was not be ruled out. The origin of the leveling off in the present experiment might be determined by improving the design of the oscillator so that the thermal resistance between the sample and the thermometer is negligible. However, the result that we wish to emphasize is that a logarithmic temperature dependence of the sound speed, as predicted by the tunneling model without TLS interactions (Eq. (1)), has been observed to a lower temperature (between 2 and 3 mK) in the present experiment than in previous experiments. The logarithmic behavior is not produced by overdriving the oscillator into the non-linear regime, and it is unlikely that any other factor could have artificially produced the logarithmic dependence. The logarithmic behavior is the intrinsic behavior down to 3 mK.

**Acknowledgements** This research is supported by NSF DMR-0457533 and 0520404. The authors would like to thank C. Enss for providing the double paddle oscillator and mounting assembly for this experiment.

## References

1. S. Hunklinger, A.K. Raychaudhuri, in *Progress in Low Temperature Physics*, ed. by D.F. Brewer (North-Holland, Amsterdam, 1986), pp. 265–344
2. P. Anderson, B. Halperin, C. Varma, *Philos. Mag.* **25**, 1 (1972)
3. J. Jäckle, L. Piché, W. Arnold, S. Hunklinger, *J. Non-Cryst. Solids* **20**, 365 (1976)
4. W.A. Phillips, *Rep. Prog. Phys.* **50**, 1657 (1987)
5. C. Enss, *Phys. B* **316–317**, 12 (2002)
6. D. Natelson, D. Rosenberg, D.D. Osheroff, *Phys. Rev. Lett.* **80**, 4689 (1998)
7. A.L. Burin, *J. Low Temp. Phys.* **100**, 309 (1995)
8. J. Classen, C. Enss, C. Bechinger, G. Weiss, S. Hunklinger, *Ann. Phys. (Leipz.)* **3**, 315 (1994)
9. J. Classen, T. Burkert, C. Enss, S. Hunklinger, *Phys. Rev. Lett.* **84**, 2176 (2000)
10. E.J. Thompson, G. Lawes, J.M. Parpia, R.O. Pohl, *Phys. Rev. Lett.* **84**, 4601 (2000)
11. E. Nazaretski, R.D. Merithew, R.O. Pohl, J.M. Parpia, *Phys. Rev. B* **71**, 144201 (2005)
12. P. Esquinazi, R. Konig, F. Pobell, *Z. Phys. B* **87**, 305 (1992)
13. C.L. Spiel, R.O. Pohl, A.T. Zehnder, *Rev. Sci. Instrum.* **72**, 1482 (2001)
14. J.M. Parpia, W.P. Kirk, P.S. Kobiela, T.L. Rhodes, Z. Olejniczak, G.N. Parker, *Rev. Sci. Instrum.* **56**, 437 (1985)
15. J. Classen, C. Enss, S. Hunklinger, *Phys. Rev. Lett.* **86**, 2480 (2001)
16. A.L. Burin, I.Ya. Polishchuk, P. Fulde, Y. Sereda, *Phys. Rev. B* **73**, 14205 (2006)
17. S. Rogge, D. Natelson, B. Tigner, D.D. Osheroff, *Phys. Rev. B* **55**, 11256 (1997)