Size and frequency dependent gas damping of nanomechanical resonators

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We examine size and frequency dependent gas damping of nanobeam resonators. We find an optimal beam width that maximizes the quality factor at atmospheric pressure, balancing the dissipation that scales with surface-to-volume ratio and dominates at small widths, against the interaction with the underlying substrate via the air that dominates the behavior of the wider devices. This latter interaction is found to affect the Knudsen number corresponding to a transition out of the molecular damping regime. We examine higher order modes and tune tension mechanically to vary the frequency of individual resonators, to resolve size and frequency effects. *© 2008 American Institute of Physics*. [DOI: 10.1063/1.2952762]

Nanomechanical resonant sensors with radio frequencies and active masses on the order of femtograms (10^{-15} g) offer impressive mass sensitivity,[1,2] maximization of which requires operation in vacuum. In atmospheric or liquid environments, the quality factor Q and consequent sensitivity is diminished.[3,4] It has been suggested that at atmospheric pressure, nanoscale structures with size on the order of the mean free path (~65 nm for air) should maintain high Q (Refs. 5 and 6) compared to larger devices. However, both size and frequency-related effects determine the dissipation, and these factors must be decoupled in order to arrive at an optimal design. We measure dissipation as a function of pressure from vacuum to 1 atm for devices in which size and frequency are independently varied to measure their separate contributions to dissipation.

We lithographically fabricated doubly clamped beams in stressed silicon nitride, and use optical drive and detection.[7–9] We first consider two resonators identical in all aspects but width w (1.5 μm and 55 nm, defined normal to the out-of-plane motion we measure), at pressures from 1 mTorr to 1 atm, in dry nitrogen. The two resonators [inset to Fig. 1(a)] have identical length and thickness (20 μm and 105 nm) and nearly identical vacuum frequency (13–14 MHz) and Q (~60 000). At low pressure, Q is dominated by intrinsic damping, independent of the pressure of the surrounding gas. With increased pressure, Q is dominated by the interaction with the gas, and behavior characteristic of the free molecular flow (fmf) regime, $Q_{gas} \propto 1/P$, is observed. At still higher pressures, resonators would crossover to a viscous regime, $Q_{gas} \propto 1/\sqrt{P}$.[10]

We explain the deviation from the fmf regime in terms of the Knudsen number, Kn, the ratio of gas mean free path λ to beam width w,[5,10,11]

$$\mathrm{Kn} = \frac{\lambda}{w} = \frac{1}{\sqrt{2}\sigma n w} = \frac{K_B T}{\sqrt{2}\sigma w P} \tag{1}$$

where σ is the scattering cross section of gas molecules, n is the molecule number density, K_B is Boltzmann's constant, T is temperature, and P is gas pressure. For $\mathrm{Kn} > 10$, beams operate in the fmf regime; $\mathrm{Kn} < 0.01$ (viscously damped regime[5,12]) is attained for wider beams than studied here. The range $.01 \leq \mathrm{Kn} \leq 10$ defines an intermediate regime. We show in Fig. 1(a) that the 55 nm wide resonator follows the fmf model closely to 1 atm ($Q \alpha P^{-1}$), and indicate the pressure where Kn=10 for each beam, beyond which deviation from the fmf model is expected.[5,13] The Q for the wider resonator exceeds the fmf expectation by 25% at ~75 Torr

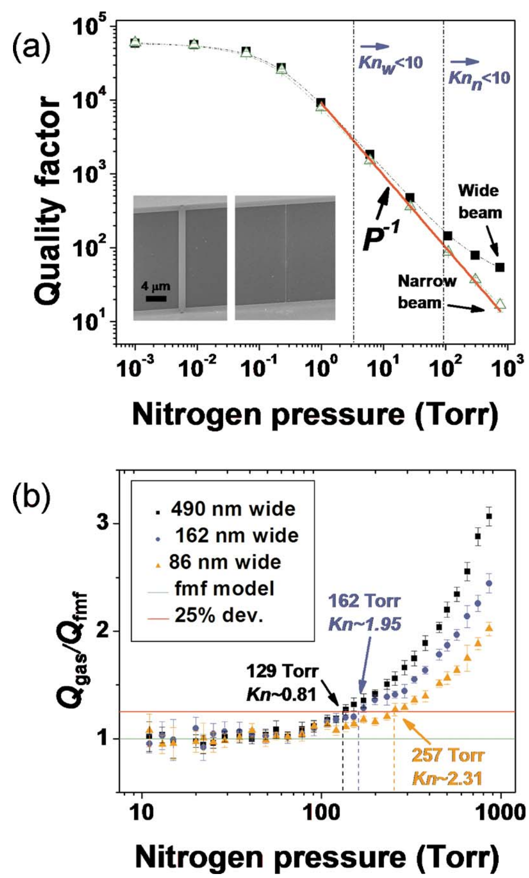

FIG. 1. (Color online) (a) Dependence of Q on N_2 pressure for two 20 μm long, 105 nm thick, high stress silicon nitride beams of similar vacuum frequency (13–14 MHz) and Q (60 000), with different widths: 55 nm and 1.5 μm, (SEM shown in the inset). The vertical lines indicate the pressure range for which the Knudsen number is <10 for the narrow (n) and wide (w) beam. (b) Q_{gas} normalized by a fmf model, as a function of N_2 pressure for 11 μm long, 140 nm thick low stress silicon nitride beams, with widths of 86, 162, and 490 nm. Deviations of 25% from the fmf model are marked at Kn numbers of 2.31, 1.95, and 0.81 (±10%), respectively.

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(Kn ~ 0.45). At atmospheric pressure, Q (1.5 μm) is ~3 $\times Q$ (55 nm) (54 versus 17). This result disagrees with the notion that narrower resonators better maintain high Q in gas, but agrees with the prediction that gas dissipation at a given pressure and frequency in an intermediate regime should scale as the surface-to-volume ratio.[14–16] We summarize the result shown in Fig. 1(a) as follows: A pair of resonators of identical length, thickness, frequency, and vacuum Q, with differing widths exhibit identical behavior in the intrinsic and fmf regimes, but depart from the fmf regime (Kn < 10) at different pressures.[5,6] With further pressure increase, the Q falls off more slowly for the viscously damped wider beam, scaling as $P^{-1/2}$, compared to the P^{-1} behavior in the fmf regime. Thus, the wider resonator exhibits higher Q at higher pressures.

To further understand the observed damping transition, we measure the Q of three beams of identical length (11 μm) and frequency (13–14 MHz), but different widths (86, 162, and 490 nm) as a function of N_2 pressure. We extract Q_{gas} using the relation

$$\frac{1}{Q_{\mathrm{tot}}} = \frac{1}{Q_{\mathrm{gas}}} + \frac{1}{Q_{\mathrm{vac}}}. \qquad (2)$$

For each beam width, Q_{gas} is fit to the fmf dependence (aP^{-1}) up to 50 Torr (below this pressure no deviation from the fmf behavior is evident). In Fig. 1(b), we plot $Q_{\mathrm{gas}}/Q_{\mathrm{fmf}}$ versus N_2 pressure. Previous work demonstrated *frequency-related* transitions between Newtonian and non-Newtonian (or inelastic and elastic) viscous damping regimes using a relaxation-time approximation, for larger scale quartz resonators, as well as nanoscale resonators similar to those studied here, over a large frequency range.[14,17,18] We note that the transitions shown in Fig. 1 are for devices of *identical frequency* and different widths. Experiments performed on a 485 nm wide, 5 μm long resonator operating at ~35 MHz, exhibit a departure from the fmf regime at comparable pressure (150 versus 129 Torr) to the lower frequency, equal width device. These transitions are therefore related to size and *not* frequency.

We mark in Fig. 1(b) the pressure at which 25% deviation from the fmf model is observed. The pressure for departure from the fmf regime *increases* with *decreasing* width, qualitatively consistent with a size-effect interpretation. However Kn at the departure from the fmf model is not constant for the three devices, instead decreasing at increasing beam width (from Kn ~ 2.3 to ~0.81 for the 86 and 490 nm wide beams). We hypothesize that this systematic shift of Kn is related to an interaction with the underlying substrate known as squeeze-film damping.

Squeeze-film damping has been widely studied in microelectromechanical systems[19–22] and leads to significantly reduced Q and deviations from simple continuum models for resonators operating in a viscous regime. In a fmf regime, Q_{sq} due to squeeze-film damping calculated from an energy-transfer model can be expressed as[11,20]

$$Q_{\mathrm{sq}} = 16\pi\left(\frac{d}{L}\right)Q_{\mathrm{free}}, \quad \text{with } Q_{\mathrm{free}} = \frac{\rho t \omega}{4}\sqrt{\frac{\pi}{2}}\sqrt{\frac{RT}{M}}\frac{1}{P} \qquad (3)$$

for beam density ρ, thickness t, gap height d, periphery length L, molecular mass M, and Q_{free} which is the Q in the absence of an underlying substrate. For narrow beams, we expect little change in squeeze film damping for varying beam width, but the dependence should become more significant for wider beams with a larger periphery term. Also, Eq. (3) is expected to yield too large an estimate for Q at small Kn (i.e., larger width).[20]

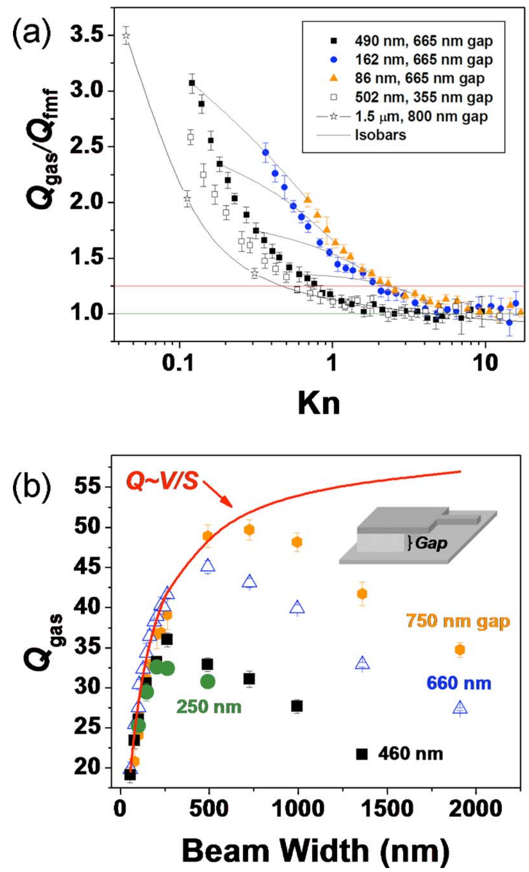

FIG. 2. (Color online) (a) $Q_{\mathrm{gas}}/Q_{\mathrm{fmf}}$ as a function of Kn for beams from Fig. 1, as well as a 502 nm wide beam suspended over a smaller (355 nm) gap. Isobars for the devices from Fig. 1(b) are plotted, from top to bottom, at 860, 568, 329, and 172 Torr. (b) Dependence of Q on beam width for 11 μm long, 140 nm thick low stress silicon nitride beams with widths ranging from 50 nm to 2 μm, and gap heights of 750, 660, 460, and 250 nm (±50 nm).

To test the hypothesis that Kn at transition is affected by squeeze-film damping, we compared the data in Fig. 1(b) from the 490 nm wide resonator with data for an identical width and frequency beam, on a different chip with a smaller gap height (355 nm, compared to 665 nm). The beam over the smaller gap transitions at a higher pressure (*smaller* Kn), at Kn ~ 0.54 compared to 0.81. In Fig. 2(a), we plot $Q_{\mathrm{gas}}/Q_{\mathrm{fmf}}$ as a function of Kn for these different gap resonators, together with devices from Fig. 1. Wider beams, or beams over a small gap, transition from the fmf regime at a smaller Kn, consistent with the transition from fmf at smaller-than-expected Kn recently reported.[6,12] We measured Q at 1 atm of air for similar devices with widths from 50 nm to 2 μm, suspended over varying gap heights. In Fig. 2(b), we see that ambient Q_{gas} peaks at a particular beam width (for a given gap height). Q for narrower beams at 1 atm. scales with volume-to-surface ratio, while Q for the intermediate and wider beams depends strongly on gap height.

In the fmf regime at a fixed pressure, Q_{free} depends linearly on resonant frequency [Eq. (3)], yielding increased Q in ambient conditions.[6,23] To quantify the frequency dependent dissipation we performed pressure scans on the funda-

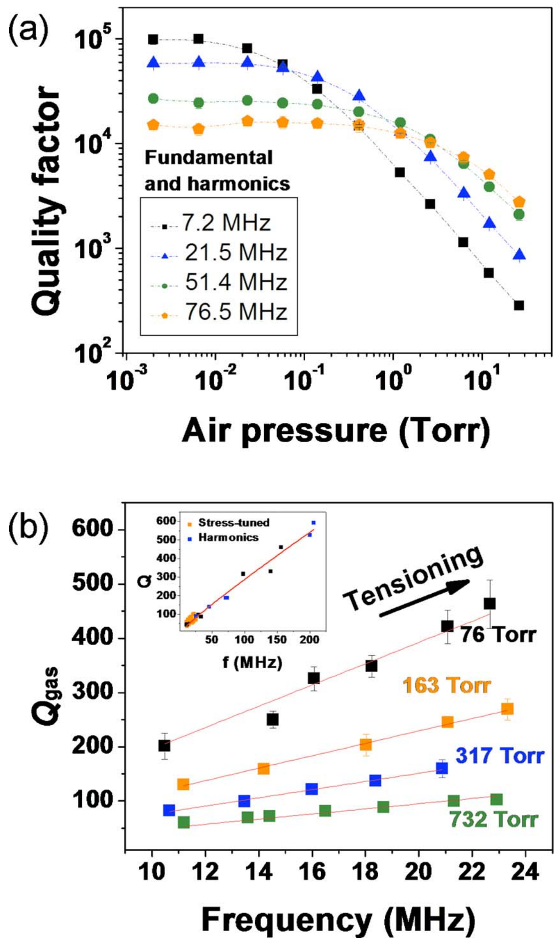

FIG. 3. (Color online) Q dependence on air pressure for a 40 μm long high stress beam, and its harmonics. (b) Q_{gas} vs. frequency, at air pressures from 0.1 to 1 atm for a 10 μm long beam frequency-tuned using a chip-bending technique. Linear fits are shown. Inset: a compilation of Q vs f at 1 atm. for a mix of devices including fundamental modes of different length beams, stress-tuned beams, and harmonics covering a frequency range from 7 to 206 MHz, exhibiting Qs from 17 to 600.

mental mode and harmonics of a 40 μm long high stress device, plotted in Fig. 3(a). The lower frequency modes have higher Q_{vac}, but lower Q_{gas} at atmospheric pressure. The source of the frequency dependence of Q_{vac} remains an open question; it is generally true for high stress silicon nitride nanostrings that Q_{vac} decreases with increasing frequency (for a constant stress value).[8] However, at higher pressure, this behavior reverses as dissipation is dominated by gas damping, with the higher frequencies displaying higher Q. The optimal frequency to maximize Q therefore depends critically on operating pressure.

To separate frequency and size effects related to gas damping we use a previously demonstrated chip-bending technique.[9] In Fig. 3(b), we show results at several pressures for a 10×1 μm^2 beam: we emphasize that we tune this resonator's frequency by varying the beam stress. For the range of pressures from 0.1 to 1 atm, Q_{gas} depends linearly on frequency, consistent with the fmf expectation despite the Kn <10 (intermediate damping regime) at atmospheric pressure. In the inset to Fig. 3(b), we compile Q versus f for a wide range of devices measured at ambient pressure. We include a mix of fundamental modes of different length beams and stress-tuned beams, as well as their harmonics, covering a frequency range from 7 to 206 MHz, and Qs from 17 to 600. This compilation illustrates that improvements in ambient Q are observed for all means of increasing resonant frequency.

We have demonstrated that dissipation at atmospheric pressure scales with surface-to-volume ratio for narrower resonators, and is dominated by squeeze-film damping for wider beams leading to a maximum Q at intermediate widths. The Knudsen number at which a transition out of a molecular flow regime occurs is found to depend on squeeze-film damping effects. We establish that the high Q associated with smaller resonators under ambient conditions stems from the higher frequency associated with shorter devices, rather than from their smaller cross section (which is approximately the mean free path). An intermediate width, or in-plane mode device could minimize dissipation associated with squeeze-film effects, whereas for mass sensing applications, the advantages of smaller mass could outweigh the disadvantages in dissipation associated with smaller resonators.

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