Dissipation Mechanisms near the Superfluid $^3$He Transition in Aerogel

A. I. Golov,1,2 D. Einzel,1,3 G. Lawes,1,4 K. Matsumoto,1,5 and J. M. Parpia1

1Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853, USA
2Schuster Laboratory, Manchester University, Manchester M13 9PL, United Kingdom
3Walther Meissner Institut für Tiertemperaturforschung, D 85748 Garching, Germany
4Los Alamos National Laboratory, Los Alamos, New Mexico 87505, USA
5Kanazawa University, Kakuma-machi, Kanazawa 920-1192, Japan

(Received 1 December 2003; published 10 May 2004)

We have investigated the dissipation ($Q^{-1}$) using the torsion pendulum technique for pure $^3$He and $^3$He-$^4$He mixtures in silica aerogel near the $^3$He superfluid transition ($T_c$) in aerogel. With pure $^3$He the $Q^{-1}$ decreases at the onset of superfluidity. When phase separated $^3$He-$^4$He mixtures are introduced into the aerogel, the $Q^{-1}$ does not decrease as rapidly and eventually increases for the highest $^3$He content. We provide a model for the related attenuation of transverse sound $\alpha$ that takes into account elastic and inelastic scattering processes and exhibits a decrease in $\alpha$ at $T_c$.

DOI: 10.1103/PhysRevLett.92.195301

The superfluid phases of liquid $^3$He are characterized by unconventional pairing correlations, reminiscent of those in the unconventionally paired heavy fermion superconductors, hole doped high-$T_c$ cuprate superconductors, the Ruddlesden-Popper system Sr$_2$RuO$_4$, and others. This unconventionality is defined through the vanishing Fermi liquid average of the pair potential (gap) $\langle \Delta_p \rangle_{FS} \equiv 0$, and is manifested by the sensitivity of thermodynamic, response and transport properties to even small concentrations of nonmagnetic impurities. Low density aerogels, dilute correlated aggregates of silica whose constituents' size is of near atomic scale (1.5 nm mean diameter) provide the opportunity to study the effects of disorder on the superfluidity of $^3$He, by itself the purest material known at low temperature.

We explore quasiparticle dissipation effects in superfluid $^3$He-B in aerogel and provide the first model for the shear viscosity of superfluid $^3$He-B in the presence of impurities. The shear viscosity, together with a Drude contribution govern the attenuation of transverse sound which is closely related to the decrease in dissipation seen in the experiment below $T_c$. We interpret this decrease as the first experimental evidence for an impurity-limited viscosity aside from metallic superconductivity.

At temperatures below 100 mK, bulk $^3$He behaves like a Fermi liquid and the transport mean free path (mfp) is determined by inelastic quasiparticle-quasiparticle scattering. Because of the restricted phase space accessible to quasiparticles as the temperature is lowered, the time between collisions, $\tau$, and the mfp both increase as $T^{-2}$. The scattering rate is manifested in transport (viscosity, thermal conductivity, spin diffusion). At the bulk superfluid transition temperature $T_{c0}$ a rapid decrease in the viscosity is observed [1–3]. The aerogel strands provide an additional mechanism for elastic scattering of quasiparticles and since the strand diameter is much smaller than the $^3$He coherence length, aerogels can be considered as an impurity added to bulk $^3$He [4]. The geometrical mfp in aerogel whose porosity (in the context of superfluid $^3$He) is typically in excess of 97% varies from ~100 nm in 97% open to ~400 nm in the 99.5% open aerogel.

Confining $^3$He to silica aerogel affects the hydrodynamics of the system. The aerogel is compressible and the $^3$He liquid motion will be coupled to that of the aerogel. At sufficiently low temperatures, the quasiparticle scattering rate will be set by the elastic scattering time from the aerogel rather than the inelastic quasiparticle-quasiparticle scattering time. The crossover temperature at which the quasiparticle-aerogel mfp is equal to the inelastic scattering length is roughly 10 mK for a typical 98% porosity sample at 20 bar [5–8].

While much attention has been focused on the phase diagram and suppression of the superfluid transition via the introduction of disorder [9–12], only a few experiments have examined transport in the disordered system at mK temperatures. High frequency acoustic experiments [5] conclusively showed that the attenuation of longitudinal sound in the normal fluid was dominated by the elastic scattering introduced by the aerogel and observed that the usual crossover from ordinary to the collisionless (zero sound) regime was eliminated as were the strong collective mode peaks observed below the superfluid transition in bulk $^3$He. Instead, the experiment showed a pronounced decrease in the attenuation below $T_c$, though no mention was made of the origin of this phenomenon. Thermal conductivity measurements show a similar effect of the disorder-limited mfp in the normal state, and exhibit a decrease in the thermal conduction at the superfluid transition, attributed to the reduced specific heat and the opening of the energy gap [13]. Spin diffusion experiments also show effects of the elastic scattering from the aerogel at low temperatures [14].

The dissipation of $^3$He in aerogel was investigated using a torsion pendulum. We examined the behavior of the dissipation ($Q^{-1}$) upon entering the superfluid state of...
$^3$He in aerogel where we found that the $Q^{-1}$ decreased markedly. When enough $^4$He was added so that the mixture phase separated, the $Q^{-1}$ did not decrease as rapidly in comparison to the behavior with pure $^3$He. Eventually (for high $^4$He content) the $Q^{-1}$ increases upon entering the superfluid state. Motivated by these observations at the superfluid transition we developed a model that describes the attenuation of transverse sound in $^3$He-filled aerogel. Ordinarily liquids support a diffusive shear mode only. However, the presence of the dilute aerogel which introduces stiffness to shear enables the propagation of a transverse sound mode.

A full description of the shear viscosity and transverse sound in superfluid $^3$He-B in aerogel (“dirty” $^3$He) will be published elsewhere [15]. Briefly, the transverse sound mode of dirty $^3$He with frequency $\omega$ and wave number $q$ is characterized by the dispersion relation

$$\omega^2 = c_1^2(\omega)q^2, \quad c_1^2(\omega) = c_{1,0}^2(1 - i\zeta(\omega)).$$  \hspace{1cm} (1)

Here $c_1(\omega)$ denotes the complex sound velocity with $\zeta(\omega)$ the dissipation. In Eq. (1) we have defined the transverse sound velocity

$$c_{1,0}^2 = \frac{\rho_s}{\rho_n + \rho_a} c_a^2,$$  \hspace{1cm} (2)

with $c_a$ the corresponding transverse sound velocity of the empty aerogel matrix. Clearly $c_{1,0}$ vanishes in the limit $\rho_a = 0$. Equation (2) describes the coupling (mass loading) of the $^3$He-B Bogoliubov quasiparticles (BQP), characterized by their normal fluid density $\rho_n$ to the aerogel system (mass density $\rho_a$). The attenuation of transverse sound is conveniently expressed as the imaginary part of the complex wave number $q$

$$\alpha(\omega) = Im q = \frac{\omega}{2 c_{1,0}} \zeta(\omega) = \frac{\omega^2}{2 c_{1,0}} \frac{\rho_n}{\rho_n + \rho_a} \left[ \frac{\eta}{\rho_n + c_{1,0}^2 \tau_1} \right].$$  \hspace{1cm} (3)

In deriving the result (3) we have generalized the collision-drag model of Higashitani et al. [16] to the superfluid B phase. The first term in curly brackets is the viscous diffusivity of the BQP, characterized by the shear viscosity $\eta$ and contains contributions from both inelastic and elastic collisions whereas the second term is a Drude type diffusivity characterizing elastic collisions with impurity transport time $\tau_1$ of the BQP with the moving impurity matrix. Note that the normal state limit of Eq. (3) agrees with the result of Higashitani et al. [16]. In the presence of both inelastic ($\eta_i$) and elastic ($\eta_e$) collisions the viscosity $\eta$ may be well approximated by the Matthiessen rule expression

$$\frac{1}{\eta} = \frac{1}{\eta_i} + \frac{1}{\eta_e} + \eta_\nu = \frac{1}{2} \rho_n V^2 \tau_1; \quad \nu = i, e.$$  \hspace{1cm} (4)

Here $\rho_n$ is the normal fluid density in the absence of Fermi liquid renormalization, $V$ is the thermal average of the BQP group velocity, and $\tau_1^2$ denotes the relaxation time of the BQP momentum current in the elastic ($\nu = e$) and the inelastic ($\nu = i$) scattering channels. The viscosity $\eta_i$ drops sharply below $T_c$ due to the nonanalytic dependence of the BQP group velocity on energy arising from the onset of the energy gap [3], and assumes a finite low $T$ limiting value of approximately $\frac{1}{2}$ of the value at $T_c$, in contrast to the slow change of the “bare” normal density $\rho_{n0}$ with temperature. The results for the impurity-limited viscosity $\eta_i$ depends on the strength of the impurity scattering (Born vs unitary limit) believed to be relevant for aerogel as the impurity system. In what follows, we adopt arguments in favor of strong impurity scattering [17] close to the unitary limit. If one neglects the pair-breaking impurity band contribution at low $T$ to $\eta_i$ [15], the viscosity $\eta_i$ assumes the simple form

$$\eta_i(T) = 2 \eta_{eN} \left[ \exp \left( \frac{\Delta(T)}{k_B T} \right) + 1 \right]^{-1},$$  \hspace{1cm} (5)

with $\eta_{eN}$ the impurity-limited viscosity of the normal state and $\Delta(T)$ the energy gap of the Balian-Werthhammer state. This form implies that $\eta_i$ drops below $T_c$, though not as fast as $\eta_\nu$, and vanishes exponentially as $\exp(-\Delta/k_BT)$ in the low temperature limit. Rewriting the attenuation as

$$\alpha(\omega) = \frac{\omega^2 \eta(T)}{2 c_{1,0}(\rho_n + \rho_a)} \left[ 1 + \frac{c_{1,0}^2}{V^2 (\rho_{n0})^2 \tau_1} \right],$$  \hspace{1cm} (6)

and applying parameters relevant to the experimental situation at $P = 20$ bar, we see that the viscosity $\eta$ is impurity limited ($\eta = \eta_i$) and provides the dominant contribution to $\alpha$ with a correction proportional to the inverse of the BQP group velocity squared. Most importantly, the attenuation drops below $T_c$, and develops a shallow plateau at intermediate temperatures originating from the Drude contribution to $\alpha$. The increase of this Drude term [the second term on the right-hand side of Eq. (6)] with decreasing temperature has its origin in the temperature dependence of $V^2 \propto T^{-0.5} T/T_c$. The transverse sound mode is diffusive in the hydrodynamic limit without aerogel and the wave number reduces to the well-known result $q = (1 + i\delta) / \delta$ with $\delta = (2\eta_B / \rho_n \omega)^{1/2}$ the viscous penetration depth. The dissipation $\alpha_1 = Im q = \delta^{-1} \left( \rho_n / \eta_B \right)^{1/2}$ decreases below $T_c$ before turning around due to the decrease of $\rho_n$. The calculated attenuation of transverse sound in (and without) 98% open aerogel below $T_c$ is shown in Fig. 1.

We now summarize a series of experiments on $^3$He and $^3$He-$^4$He mixtures in aerogel and focus on the dissipation in the $^3$He-aerogel system at $T_c$. The shift in resonant frequency of the cell identifies $T_c$ as well as the superfluid fraction, while the dissipation is proportional to the drive needed to maintain a constant amplitude of motion. The dissipation could be tracked across the normal-superfluid boundary as described below.
FIG. 1. The calculated transverse sound attenuation vs $T/T_c$. The attenuation (with aerogel: total) decreases sharply at $T_c$. The dominant contribution to the attenuation (“viscous”) shows a strong decrease at $T_c$. The elastic contribution (“Drude”) shows a slow increase originating in the square of the inverse BQP group velocity. Also shown is the behavior of the attenuation in the absence of aerogel, i.e., the limiting behavior if the $^3$He eliminates elastic scattering.

Figure 2 shows the dissipation as a function of the $^3$He pressure while the cell is maintained at a fixed temperature of 0.3 mK. The cell was filled to 14 bar, then cooled to 0.3 mK. The pressure of the $^3$He was slowly reduced, and the temperature held fixed by demagnetization of the nuclear cooling stage as necessary [18]. At this temperature there is no superfluid $^3$He at pressures below 6.2 bar. At higher pressures, the superfluid fraction increases and we are able to determine $T_c$ and the amount of superfluid present using the period shift of the oscillator, independent of the dissipation measurement.

Below 6.2 bar, the $^3$He is in the normal state and is nearly locked to the aerogel. The $Q^{-1}$ is independent of pressure over this limited range, since all the quantities depend only weakly on pressure. When the pressure is high enough for superfluidity to develop in the aerogel, there is a change in the dissipation. The observed decrease in $Q^{-1}$ [related to $\alpha$ in Eq. (6)] is consistent with the expected decrease in the impurity-limited viscosity $\eta_i$ according to Eq. (5). The various sharp peaks in $Q^{-1}$ mark crossings of the second-sound-like “slow mode” with the frequency of the torsional oscillator [4,19,20]. Other than the viscosity, all the quantities in Eq. (6) vary slowly with pressure, and thus cannot be responsible for the rapid decrease in dissipation across the superfluid transition.

Subsequent experiments which investigated the $Q^{-1}$ of $^3$He and $^3$He-$^4$He mixtures in aerogel with a $\approx 877$ Hz torsion pendulum also used a 98% porosity sample [21]. This experiment investigated the effect of adding $^4$He to the $^3$He on $T_c$, $\rho_s$, and dissipation. A small gap ($\approx 0.1$ mm) between the aerogel and the cell contained bulk fluid that contributed significantly to the $Q^{-1}$.

The $Q^{-1}$ results are shown in Fig. 3, with curves offset vertically for clarity. Above the bulk superfluid transition, $T_{c0}$, the $Q^{-1}$ is almost flat, reflecting the almost constant viscosity of the $^3$He in aerogel. The weak temperature dependence is due to the bulk liquid that is nearly clamped to the oscillator’s motion. At $T_{c0}$ ($\approx 2.27$ mK) the bulk viscosity $\eta$ drops abruptly and produces a sharp increase in the $Q^{-1}$ because of the decrease in the viscous penetration depth $\delta$ [see paragraph after Eq. (6)]. Below $T_{c0}$, but above the aerogel $T_c$, the $Q^{-1}$ changes slowly due to the nearly constant $\delta$ resulting from the combination of decreases in both $\eta$ and $\rho_s$ [22].

Just below $T_c$ of $^3$He in aerogel for pure $^3$He (actually the very similar case of 2% $^4$He that forms a solid film on the aerogel), the $Q^{-1}$ drops in the narrow temperature range $0.9T_c \leq T \leq T_c$ because of the decrease in the
impurity-limited \( \eta \) in a manner similar to that seen in Fig. 2. The behavior between \( T_d \) and \( T_c \) is not affected by the \( ^4\text{He} \), but the signature below \( T_c \) is strikingly different. For intermediate concentrations of \( ^4\text{He} \), the \( Q^{-1} \) is nearly constant at \( T_c \) and decreases gradually on cooling. For higher \( ^4\text{He} \) content, the \( Q^{-1} \) increases sharply at \( T_c \), and continues to increase with cooling. The numerous subsidiary features in the \( Q^{-1} \) below \( T_c \) are due to losses to various coupled sound modes present in mixtures \([4,20,23]\).

Our model does not explicitly take into account the presence of the \( ^4\text{He} \) on the aerogel. Phase separation occurs for \( ^4\text{He} \) concentrations \( x_4 \geq 11\% \), a superfluid \( ^4\text{He} \)-rich mixture covers the strands and pure \( ^3\text{He} \) occupies the remaining volume \([23–25]\). The effect of the \( ^4\text{He} \) is twofold: first, the \( ^4\text{He} \)-rich phase covering the aerogel strands enhances the mass of the normal component, \( \rho_n + \rho_a \), entrained to the oscillator while the shear stiffness, \( c_s^2 \rho_a \), is unchanged; second, the \( ^4\text{He} \)-rich phase occupies the smallest voids (most rough regions) of the fractal aerogel while the \( ^3\text{He} \) phase occupies the biggest voids in contact with the smooth interface between the \( ^3\text{He} \) and \( ^4\text{He} \)-rich phases. Both effects modify the dissipation, with the latter case being dominant. In this regime the BQP do not interact directly with the aerogel, but only with the helium mixture covering the strands.

Quasiparticle scattering continuously changes from diffuse to specular when the thickness of the \( ^4\text{He} \)-rich phase on aerogel increases \([21,26,27]\). The experimental phenomenology displayed by the introduction of \( ^4\text{He} \) can be interpreted in the spirit of Fig. 1 which shows a reduction of dissipation below \( T_{cr} \) that accompanies the decrease in impurity-limited viscosity [Eq. (6)]. The presence of elastic scattering from the aerogel provides the mechanism by which momentum can be coupled to the \( ^3\text{He} \), in effect relaxing the momentum conservation property. In the opposite limit (no aerogel) one expects the diffusive shear mode to be present as a consequence of momentum conservation, with a dissipation \( \propto \delta^{-1} \), which increases below \( T_{cr} \). The presence of the \( ^4\text{He} \)-rich phase can be envisioned as resulting in a continuous restoration of momentum conservation in the quasiparticle system, and enabling relative motion between the \( ^3\text{He} \) and aerogel. It is important to note that the small fraction of bulk \( ^3\text{He} \) in the cell (see Fig. 3) displays an increase in the dissipation in accord with this interpretation \([22]\). An inclusion of the detailed effects of specularity is in principle possible but is beyond the scope of this Letter.

To summarize, our torsion oscillator experiments on superfluid \( ^3\text{He} \) in aerogel exhibit a strong temperature dependence of the dissipation near \( T_c \) that is linked (for the first time) to the quasiparticle shear viscosity in the presence of impurities (the aerogel matrix). Because of the opening of an energy gap below \( T_c \) the viscosity decreases in a way which depends on the properties of the aerogel matrix. We propose a simple model that includes both the inelastic and elastic scattering of Bogoliubov quasiparticles in the presence of aerogel. The assumption that the aerogel system supports a transverse sound mode to which the quasiparticle system may be coupled, allows for a qualitative understanding of the observed decrease in dissipation below \( T_c \) in superfluid \( ^3\text{He} \). The increase of dissipation observed below \( T_c \) in \( ^3\text{He} \)-\( ^4\text{He} \) mixtures is ascribed to a reduction of the relative importance of impurity scattering due to increased specularity at the \( ^4\text{He} \) covered aerogel.

We acknowledge early contributions by E. Thuneberg. The research was supported by NSF DMR-0202113, 0071630, and NATO SA(PSTCLG.979379)6993/FP. A. I. Golov was supported by EPSRC GR/N35113.