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Estimate of the gap parameter for superfluid ^{3}He in aerogel

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We infer the magnitude of the superfluid gap for ^{3}He in aerogel from measurements of the normal fluid density as a function of temperature. We compute the effective Yosida function for two different aerogel samples over a range of pressures. We find that the suppression factor of the zero-temperature superfluid gap, scaled by k_BT_c, ranges from 0.5 to 0.8 as compared to the pressure-dependent weak-coupling plus gap, with the scaling factor dependent on the suppression of T_c relative to the bulk value.

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The magnitude of the gap in the quasiparticle energy spectrum is an important parameter in characterizing the superfluid phase of ^{3}He (Ref. 1) and, in particular, is directly related to the pairing amplitude. Because the gap shifts the quasiparticle energy levels, it plays a crucial role in determining the thermodynamic properties of the system. Many experiments, including NMR and heat capacity measurements, explicitly probe the magnitude of the superfluid gap. Furthermore, the gap is often a straightforward parameter to compute in many models,[2,3] so it can be used to compare between theory and experiment. In this paper we will compare our parametrization of the magnitude of the superfluid gap of ^{3}He in aerogel obtained from superfluid density measurements in 98% open and 99.5% open aerogels,[4,5] together with unpublished results, to bulk values of the gap in order to elucidate the effects of the scattering impurities on the superfluid pairing amplitude.

The gap for the *B* phase of superfluid ^{3}He is isotropic and can be understood to first approximation using the BCS theory for *s*-wave superconductors.[1] However, the additional degrees of freedom in the order parameter alter the pairing interaction and cause a modification to the gap. Experimentally, the gap parameter is found to scale as a function of pressure and temperature.[6,7] The weak-coupling plus (WCP+) model proposes that the BCS gap is scaled by pressure-dependent strong-coupling terms,[8] with a temperature dependence that reduces these corrections for small values of T/T_c. The WCP+ calculations of the superfluid gap have been found to agree well with torsional oscillator[7] and heat capacity measurements[9] on the *B*-phase gap in bulk ^{3}He.

^{3}He in aerogel has been shown to undergo a superfluid transition when confined to a porous aerogel, a very dilute network of $\approx$3-nm-diameter silica strands. Measurements using torsional oscillators,[4] NMR,[11] high-[12] and low-[13] frequency sound, and heat capacity[14] have all found that ^{3}He in aerogel undergoes a superfluid transition with reduced values for the superfluid transition temperature (T_c) and superfluid density (ρ_s) as compared to bulk ^{3}He. There has also been intense theoretical interest in understanding the behavior of ^{3}He in an aerogel impurity.[2,15,16] It should be emphasized that in the general case of a disordered superfluid, the gap in the energy spectrum is no longer equivalent to the pairing amplitude. This is strikingly demonstrated in recent work which predicts the existence of gapless superfluidity for ^{3}He in aerogel over a certain region in parameter space.[10] Nevertheless, in order to simplify our discussion and emphasize that we compare our data to models for *bulk* ^{3}He rather than impure ^{3}He, we will refer to the magnitude of the superfluid order parameter as the gap throughout this work.

Estimates of the gap based on direct NMR and heat capacity measurements show that the superfluid gap for ^{3}He in aerogel is suppressed compared to the bulk value. The zero-temperature gap estimated from NMR measurements is roughly $\Delta_{aerogel} \approx 0.5\Delta_{bulk}$,[17] which agrees well with heat capacity measurements.[14] By contrast, recent experiments on superfluid ^{3}He in aerogel using an oscillating disk of aerogel in a ^{3}He bath find that the gap suppression measured by the *A*-*B* transition's magnetic field dependence[18] is consistent with a simple scaling of the gap as predicted by the homogeneous scattering model for superfluid ^{3}He in aerogel. This is a substantially smaller reduction than inferred from the earlier measurements. Calculations for *d*-wave superconductors with impurity scattering find that the average gap does not simply scale with T_c.[19]

The effect of impurity scattering on superfluid ^{3}He in aerogel is expressed in models in terms of the parameter l/ξ_0, where l is an impurity mean free path and ξ_0 the superfluid coherence length.[2] Comparisons between experimental data and these models have generally relied on varying l as a fitting parameter, which has been found to exhibit pressure dependence.[2,15] Experimentally, the ratio l/ξ_0 is most easily altered by varying the pressure, which in turn changes ξ_0. However, the effective l may also be varied by changing the density of the aerogel, and we present our analysis of results from 99.5% aerogel which have not been discussed in earlier references.[2,15] It is well known from experiments on bulk ^{3}He that strong-coupling corrections affect the superfluid pairing at high pressures;[8] these corrections are not accounted for in current models for ^{3}He in aerogel.[15] Our goal in this paper is to extract an estimate of the pairing amplitudes for ^{3}He in aerogel from our measurements on the superfluid density in two different aerogel samples. We first compare these with bulk values for the BCS superfluid gap and then to values which explicitly incorporate strong-coupling corrections in order to motivate a better understanding of the features which need to be incorporated into a model for impurity scattering of ^{3}He in aerogel which includes strong-coupling corrections.

The experiments consisted of a torsional oscillator with the aerogel occupying the main inertial mass. The cell was mounted on a dilution cryostat with a PrNi_5 nuclear demagnetization stage, having a base temperature of 0.7 mK. The sample temperature was measured using a lanthanum-diluted cerium magnesium nitrate thermometer calibrated to within 10 μK using the specific heat anomaly in bulk superfluid ^{3}He.

By monitoring the temperature-dependent period shift of the torsional oscillator below T_c and comparing this to the change in period on filling the cell with ^{3}He, we were able to measure the fraction of ^{3}He which decoupled from the torsion head. After correcting for tortuosity, we converted this period shift into a superfluid density. We measured the superfluid density as a function of temperature for several different pressures for the two aerogel samples. The aerogel samples had porosities of 98% and 99.5%, covering a factor of 4 in impurity density. The superfluid transition of ^{3}He in these particular samples has been discussed previously,[4,5] and portions of the superfluid density data used in this paper are presented in these earlier references.

Superfluid ^{3}He in aerogel at low fields and pressures is thought to be in the B phase with an isotropic gap.[20] By comparing the temperature dependence of the bare normal fluid density (ρ_n^b, defined below) derived from the torsional oscillator measurements to the calculated Yosida function, a measure of the density of thermal excitations on the Fermi sphere, we are able to estimate the suppression of the superfluid gap due to impurity scattering by the aerogel.

There is a pressure-dependent adjustment to the normal fluid density arising from Fermi liquid corrections to the effective quasiparticle mass. In a manner similar to that described in earlier references,[3,6] we can strip away these Fermi corrections by defining the bare normal fluid density as

$$\frac{\rho_n^b}{\rho}=\frac{\dfrac{\rho_n}{\rho}}{1+\left(\dfrac{F_1}{3}\right)\left(1-\dfrac{\rho_n}{\rho}\right)}, \tag{1}$$

where ρ_n/ρ is the normal fluid density measured by the torsional oscillator experiment and F_1 is the pressure-dependent Landau parameter.

Calculating the Yosida function, which is equivalent to ρ_n^b, first involves computing the magnitude of the gap and then using this energy to find the density of thermally excited quasiparticles. The isotropic temperature-dependent Yosida function[21]

$$Y(T)=\int_0^\infty d\epsilon_k \beta \,\mathrm{sech}^2 \frac{1}{2}\beta E_k \tag{2}$$

depends explicitly on the gap parameter $\Delta(T)$ through the quasiparticle dispersion equation

$$E_k=+\sqrt{\epsilon_k^2+[\Delta(T)]^2}. \tag{3}$$

To calculate the magnitude of the gap parameter, we use the interpolation formula[22]

$$\frac{\Delta(T)}{k_BT_c}=\frac{\Delta(T=0)}{k_BT_c}\tanh\frac{\pi}{\delta_{sc}}\left[\frac{2}{3}\frac{\Delta C}{C_N}\left(\frac{T_c}{T}-1\right)\right]^{1/2}, \tag{4}$$

where $\delta_{sc}=\Delta(T=0)/k_BT_c$ parametrizes the strong-coupling corrections and $\Delta C/C_N$ is the specific heat discontinuity. It should be pointed out that using Eq. (4) in order to compute the Yosida function expresses the magnitude of the gap in terms of the dimensionless parameter Δ/k_BT_c. Because the superfluid transition temperature for ^{3}He in aerogel, T_c, is reduced relative to the bulk ^{3}He superfluid transition temperature T_{c0}, there will be an additional term T_c/T_{c0} in the suppression of the magnitude of the gap parameter.

It is convenient to factor out the explicit dependence on the transition temperature of the zero-temperature gap by defining a parameter δ (motivated by the definition of the strong-coupling parameter δ_{sc} in Ref. 22), as

$$\delta=\frac{\Delta(T=0)}{k_BT_c}, \tag{5}$$

where T_c is the superfluid transition temperature. Then, for BCS systems, δ will always equal 1.76, regardless of any change in the magnitude of the zero-temperature gap due to a suppression of T_c. In practice, we find that the value of δ for superfluid ^{3}He in aerogel shows a systematic dependence on T_c/T_{c0} (where T_{c0} is the bulk superfluid transition temperature).

We first compare the bare normal fluid density data from the torsional oscillators to the Yosida function arising from a pressure-independent BCS gap. The BCS gap is computed from Eq. (4) using $\delta_{sc}=1.76$ and $\Delta C/C_N=1.43$. The temperature-dependent quasiparticle energy, found using Eq. (3), was substituted into the integrand in Eq. (2) which was numerically integrated with appropriate energy cutoffs. In order to investigate a suppression of the gap by a simple scaling factor, we computed the Yosida function for a range of BCS gap parameters reduced by a multiplicative coefficient ranging from 1 to 0.53.

Figure 1 shows the comparison between the bare normal fluid density from the torsional oscillator (TO) experiments and the calculated scaled BCS Yosida functions. The upper plot is for the 98% porosity aerogel, while the lower plot is for the 99.5% sample. The data are shown for three pressures (25, 10, and 2 bars) where the superfluid transition temperature is determined by the onset of the period shift in the TO. Although ρ_n^b/ρ shows some deviations from the calculated Yosida functions close to T_c, it is possible to estimate an "effective" scaled BCS gap over a relatively large temperature range for each set of data. On the basis of Fig. 1 we see evidence for a pressure-dependent suppression of the superfluid gap (scaled by T_c) for ^{3}He in aerogel.

The curvature of the bare normal fluid density is different from the linear behavior of the Yosida function close to T_c. This feature may be more clear in Fig. 2. The temperature dependence of the superfluid fraction of ^{3}He in aerogel does not follow that of the bulk fluid at any pressure. In particular, Porto and Parpia found that the superfluid density followed a power law behavior over a large temperature range.[4] This deviation could indicate some change in the pairing mecha-

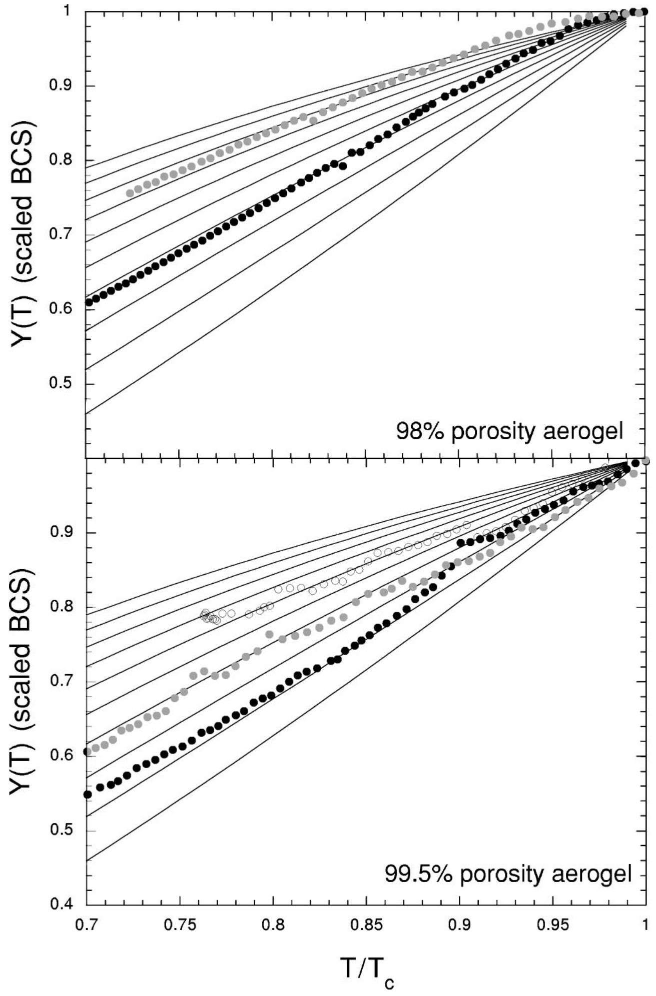

FIG. 1. Comparison of the scaled BCS Yosida function with ρ_n^b/ρ. The upper plot shows the result for a 98% porosity aerogel while the lower was taken with a 99.5% sample. The solid symbols are at 25 bars of pressure, the gray symbols at 10 bars, and the open symbols at 2.5 bars. The Yosida functions are plotted as solid lines, with BCS gap scaling factors of (from bottom to top) 1, 0.91, 0.83, 0.77, 0.71, 0.67, 0.63, 0.59, 0.56, and 0.53.

nism in the presence of impurity scattering. A quantitative analysis of this discrepancy is hindered by the presence of sound modes crossing the TO resonant frequency close to T_c which affects the measurement of ρ_n^b. This small discrepancy between the Yosida function and ρ_n^b is incorporated into the error bars in Fig. 3.

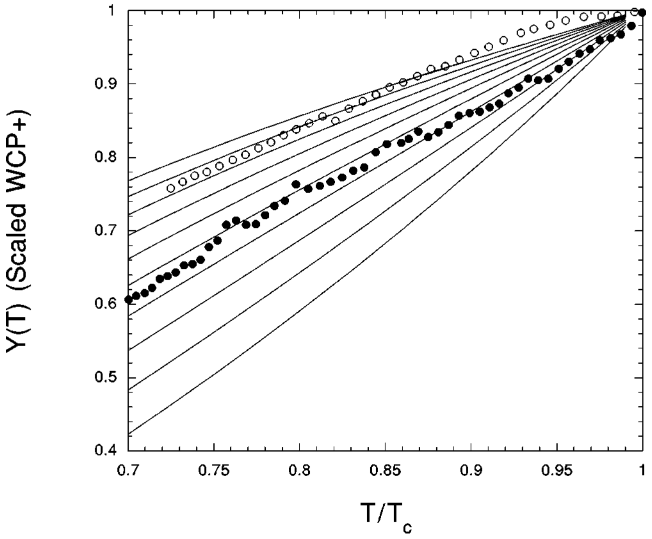

FIG. 2. The temperature-dependent Yosida function computed from the scaled WCP+ gap at a pressure of 10 bars. The WCP+ gap is scaled by the same factors as in Fig. 1, with the lower curve corresponding to a scaling factor of 1.0 and the upper curve a scaling factor of 0.53. Also plotted are the bare normal fluid densities for a 99.5% aerogel sample (solid symbols) and a 98% aerogel sample (open symbols), both at 10 bar.

Rather than compare the bare normal fluid density to the BCS gap, we can also compare these data to the Yosida function computed from the WCP+ gap, which yields a more accurate estimate of the gap for *B*-phase bulk ^{3}He. The pressure-dependent WCP+ gap was computed using the WCP+ parameters[23] for δ_{sc} and $\Delta C/C_N$ in Eq. (4). The values of δ_{sc} used in computing the WCP+ gap vary from 1.774 at 0 bar to 1.866 at 34 bars. Calculating the Yosida functions for a set of scaled WCP+ gaps led to a family of curves similar to those for the scaled Yosida function in Fig. 1 for each pressure. Figure 2 shows a specific example of this at 10 bars. This plot shows the temperature dependence of the Yosida function from the scaled WCP+ gap, with scaling factors between 1 and 0.53 as in Fig. 1, plotted with the normal fluid density for ^{3}He in both 98% and 99.5% aerogel samples. Notice that at a fixed temperature the Yosida function from the WCP+ gap is smaller than the Yosida function

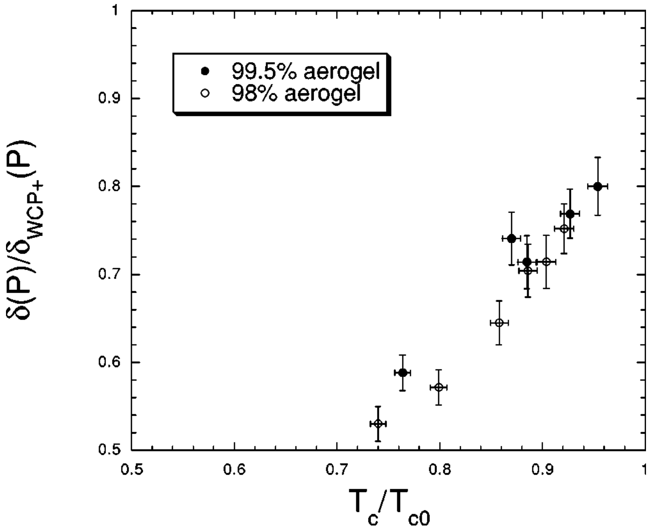

FIG. 3. Scaling factor for the WCP+ gap, expressed in terms of δ/δ_{WCP+}, which gives the best fit Yosida function away from T_c plotted vs T_c/T_{c0}. Here δ is defined in Eq. (5), and the pressure dependence due to strong-coupling corrections is explicitly stated. The solid symbols are from the 99.5% porosity aerogel sample, while the open symbols are from the 98% porosity sample. The error bars in the scaling factor represent the uncertainty in choosing the best fit Yosida function, while the errors in T_c/T_{c0} result from the determination of T_c.

from the BCS gap. By comparing the bare normal fluid density at some pressure to these WCP+ Yosida functions at the same pressure, we estimated the effective suppression in the WCP+ gap which gave the best fit to the experimental data.

We plot the suppression in the effective WCP+ gap, in terms of δ/δ_{WCP+}, which produces the best fit Yosida function versus T_c/T_{c0} for a range of pressures in both aerogel samples in Fig. 3. The error bars show the estimated range in the gap scaling factor based on uncertainty in choosing the best fit Yosida function. We see evidence for a systematic dependence of the scaling factor for δ, defined in Eq. (5), on T_c/T_{c0}. The magnitude of the WCP+ gap parameter scaled by k_BT_c depends strongly on pressure for a given aerogel sample (since T_c/T_{c0} depends on pressure[4]). It is important to note that by plotting the suppression of δ rather than the suppression of the $T=0$ gap we have eliminated the explicit dependence on T_c, and by computing the gap using the WCP+ parameters we have eliminated the pressure-dependent strong-coupling corrections. This means that the suppression of δ shown in Fig. 3 represents variations of the gap parameter arising solely from impurity scattering from the aerogel sample.

The magnitude of the gap scaling factor δ/δ_{WCP+} depends on the suppression of T_c and not on the specific aerogel sample. Since T_c has been shown to depend on the microstructure of the aerogel,[5] the differences between the two aerogel samples are parametrized simply by the suppression of T_c. When the additional correction to the suppression of the gap arising from the reduction of T_c due to impurity scattering is included, the magnitude of the zero-temperature gap in superfluid ^{3}He at a fixed pressure is reduced by a factor of ≈ 0.4 to ≈ 0.76 (depending on pressure) in aerogel as compared to bulk fluid.

We have estimated the magnitude of the superfluid gap for ^{3}He in aerogel by comparing the bare normal fluid density to Yosida functions calculated using the BCS and WCP+ gaps. We find evidence that the gap is reduced by a scaling factor which depends on pressure, beyond the variations due to the normal strong-coupling corrections. This immediately suggests that any successful model for ^{3}He in aerogel must simulate this T_c-dependent suppression of the gap. It should be pointed out that the estimates for gap discussed in this paper were based on ρ_n^b over a limited temperature range, between $T=0.7T_c$ and $T=0.95T_c$, as compared to the much broader temperature range investigated previously. Our estimates for the suppression of the gap agree well with previous heat capacity and NMR measurements (for the appropriate values of T_c/T_{c0}), and a T_c-dependent suppression of the gap is qualitatively in agreement with predictions made for *d*-wave superconductors with impurity scattering.[19]

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