NON-DEGENERATE NANOMECHANICAL PARAMETRIC AMPLIFIER

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ABSTRACT

We present a nanomechanical parametric amplifier system, which consists of two electrostatically coupled resonators, and operates using 3 frequencies (non-degenerate amplifier). The use of 3 frequencies eliminates phase sensitivity between the signal and pump voltage. We observed the amplification of small signals by up to 43 dB. The width of the response of the resonator is tuned by varying the pump voltage. The resonant frequencies of the individual oscillators can be slightly tuned by adjusting the bias voltages. To our knowledge, this is the first reported mechanical parametric amplification scheme using two separate resonators.

INTRODUCTION

The mechanical properties of micro- and nanomechanical systems are of interest from both fundamental and application standpoints. Recent developments include the detection of small charges and forces, [1,2] and mesoscopic studies of vortex physics in superconductors. [3] Small mechanical structures have also been used to gain understanding into mechanical losses on the sub-micron scale, surface and near surface effects, and other fundamental phenomena such as coupling with lattice vibrations and interaction with electromagnetic fields.

Dynamical properties of mechanical resonators can be utilized to create a parametric amplifier. Parametric amplification has long been used as a technique for making a low noise electronic amplifier. The amplification of the applied signal is done by making use of the energy from the pumping action. Parametric amplifiers with a variable-capacitance main-oscillator semiconductor diode are used in radar tracking and communications between Earth stations, satellites, and deep-space stations. The noise temperature of cooled amplifiers is in the range of 20 to 30 K, and the gains are up to 40 dB. [4,5] This type of amplification is also widely used in optics as well as in electronic traveling wave applications. Parametric amplification was first observed in a micro-mechanical system by Rugar and Gutter in 1991. [6] They amplified the signal by applying the pump voltage at double frequency, making a degenerate amplifier. Similar effects were observed in cantilever resonators.[7, 8] We observed the same effects in torsional resonators. [9] The usefulness of the degenerate amplifier may be limited due to phase sensitivity. It is often undesirable to have an amplifier that requires a specific phase of the signal that is to be amplified. In an alternative design, a parametric amplifier can use two resonant circuits which are parametrically coupled through a time-varying capacitor that is modulated at the sum frequency. This type of amplifier (non-degenerate) does not have any phase dependence.[4] Here we report on the non-degenerate parametric amplifier system, which consists of two electrostatically coupled resonators (see Fig. 1). To our knowledge, this is the first reported parametric amplification scheme using two separate mechanical resonators. This system operates using three frequencies, and does not suffer from phase dependence. The stability of the oscillations is also improved due to increase of the effective quality factor.

THEORY

The expected behavior of a non-degenerate parametric amplifier has been derived previously. [4] For two systems described by equations:

$$\ddot{\varphi}_1 + \omega_1^2\varphi_1 + \frac{\omega_1}{Q_1}\dot{\varphi}_1 = \tau_{ext,1} + k'(t)(\varphi_1 - \varphi_2) \qquad (1)$$

$$\ddot{\varphi}_2 + \omega_2^2\varphi_2 + \frac{\omega_2}{Q_2}\dot{\varphi}_2 = \tau_{ext,2} + k'(t)(\varphi_2 - \varphi_1) \qquad (2)$$

where $\varphi_{1,2}$ are the coordinates, $Q_{1,2}$ are the Q-factors, $\tau_{ext\ 1,2}$ are the external torques, $\omega_{1,2}$ are the angular frequencies or the two resonators; and $k'(t)$ is the oscillating coupling spring constant.

In the linear approximation, the force between oscillators is proportional to the angular position difference of the two resonators, and voltage difference squared. This coupling changes the effective restoring coefficient and serves as the equivalent of the time-varying capacitor used in electronic parametric amplifiers. We can change the above equations into two pairs of first-order differential equations through the use of the normal mode approach described by Louisell. [4] A time dependent spring constant oscillating at $\omega_1+\omega_2$ results in amplification of a force applied to one of the springs. The amplitude of the motion is amplified by phase-independent gain factor [10]:

$$G = \frac{1}{\left(1+\frac{k'}{2}\sqrt{\frac{Q_1Q_2}{k_1k_2}}\right)}\frac{1}{\left(1-\frac{k'}{2}\sqrt{\frac{Q_1Q_2}{k_1k_2}}\right)} \qquad (3)$$

The resonator will have a Lorentzian response with improved effective Q-factor. This is an amplifier that also acts as a filter. Not only is it useful for amplifying small signals, but it also improves the stability of an oscillating system.

EXPERIMENTAL DETAILS

The fabrication, electrostatic actuation, and optical detection of these devices have been described previously. [11] The released structures are produced using electron beam lithography on silicon-on-insulator wafers consisting of *1 m* -thick oxide buried underneath of 250 nm of single crystal (100) silicon. The devices are not metallized; we have enough conductivity by wirebonding contact wires near the paddles. The devices are operated in a vacuum 10^{-4} Torr at room temperature. The mechanical motion is detected using optical interferometry. The motion of the structures

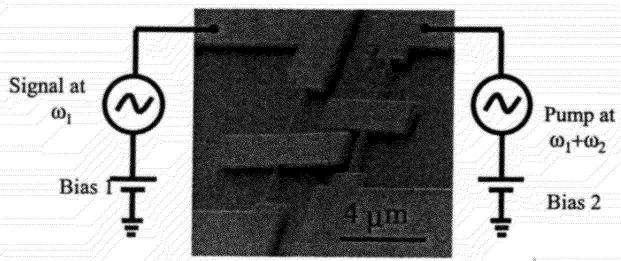

Figure 1. Scanning electron micrograph of coupled resonators with connections.

modulates the reflected laser light, and is detected by a AC-coupled photoreceiver. The laser can be focused down to a few micron spot size, which allows observation of motion of a specific resonator. The frequency of the electrical driving signal is swept near resonance, and the mechanical response curves are measured. Peak amplitude and width are measured by fitting a Lorentzian to a response curve.

We studied the device consisting of two torsional resonators, positioned in close proximity of each other (see Fig. 1). The resonators consist of a rectangular paddle suspended by narrow beams 200 nm in width. In earlier work, we have shown that the amplitude of the torsional mode can be substantially increased by introducing a small asymmetry in the supports. [12] The paddles used in this study had 0.5μm asymmetry (less than 10% of paddle width). The torsional resonators had f_1=1.39 MHz and f_2=2.66 MHz resonant frequencies. These frequencies can be varied by adjusting the device dimensions. Initially we apply a very small driving voltage to the first resonator, and observe its mechanical response (see Fig. 2a). Then the pump at the sum of the resonator frequencies is applied to the second resonator. As the pump is turned on, this voltage is greatly amplified (Fig. 2b). The small shift in the resonant frequency is due to extra DC voltage (which comes from time-averaging of squared AC voltage) being applied to the resonator. As seen from Eq. 3, there is no limit to the gain coefficient achievable with such system. However, applying high gains will also amplify thermomechanical noise, and the system with start self-oscillating. We observed maximum amplification of $2.1\text{x}10^4$ times (or 43 dB), before the system starts self-resonating. Note the significant increase in the effective quality factor of the resonator, which reduces phase noise and improves the frequency stability of the system. As in some other amplifiers, the gain has asymptotic behavior. The system is very stable only up to 20 dB amplification, depending on the stability of the support electronics. Amplitude-limiting feedback loops may improve stability and allow reliable use of higher gains.

The measured amplification coefficient as a function of the pump voltage is shown in Fig. 3. Assuming the interaction force between oscillators is proportional to the square of the voltage, and using Eq. 3, we arrive at good agreement with the experimental observation. Ignoring of higher order effects is the likely cause of the sharper rise of the experimental curve.

The width of the response of the resonator is tuned by varying the pump voltage. The resonant frequencies of the individual oscillators can be slightly tuned by adjusting the bias voltages. Qualitatively similar effects were observed with devices of other geometries. In general, the closer the resonators are to each other, the smaller the AC pumping voltage when the parametric amplification starts. It should be possible to decrease this voltage substantially
by positioning the resonators on top of each other using multi-layer lithography, which would greatly increase coupling.

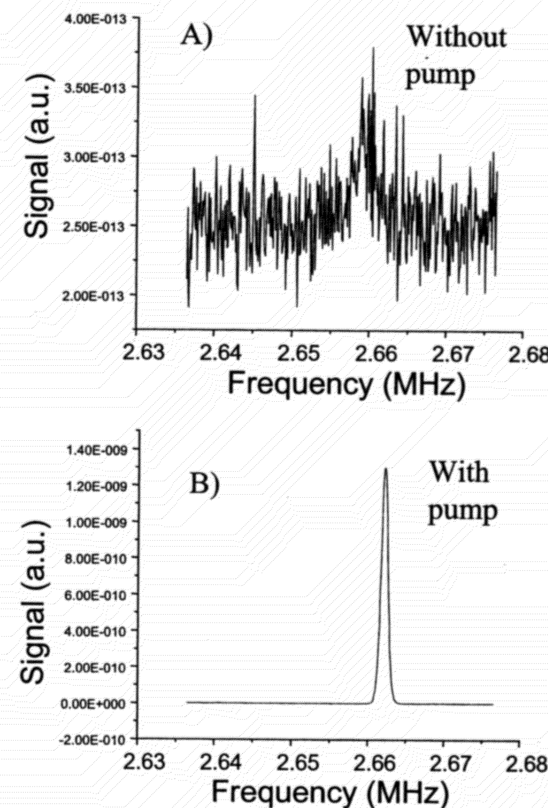

Figure 2. Mechanical response (a) without and (b) with the pumping.

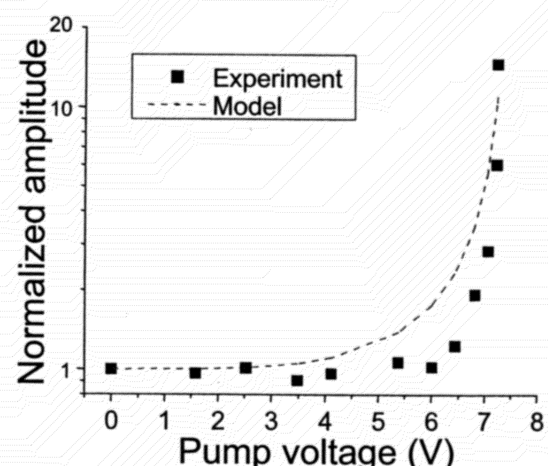

Figure 3. Measured and modeled normalized amplitude as a function of the pump voltage.

CONCLUSIONS

Overall, we present the first nanomechanical non-degenerate parametric amplification system, working on three frequencies. It shows large phase-independent amplification of small mechanical motion. It can be used for improving the detection sensitivity of resonant mechanical sensors such as cantilever mass sensors, magnetic sensors, AFM. The method can be used for boosting the signal before electronics processing, or for application in harsh environments unsuitable for regular electronics.

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