Optically pumped parametric amplification for micromechanical oscillators

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Micromechanical oscillators in the rf range were fabricated in the form of silicon discs supported by a SiO_2 pillar at the disk center. A low-power laser beam, ($P_{\mathrm{laser}}\sim 100\ \mu$W), focused at the periphery of the disk, causes a significant change of the effective spring constant producing a frequency shift, $\Delta f(\Delta f/f\sim 10^{-4})$. The high quality factor, Q, of the disk oscillator ($Q\sim 10^4$) allows us to realize parametric amplification of the disk's vibrations through a double frequency modulation of the laser power. An amplitude gain of up to 30 was demonstrated, with further increase limited by nonlinear behavior and self-generation. Phase dependence, inherent in degenerate parametric amplification, was also observed. Using this technique, the sensitivity of detection of a small force is greatly enhanced. [DOI: 10.1063/1.1371248]

Micromechanical oscillators represent an important class of microelectromechanical system (MEMS). A number of applications, such as force microscopy,[1,2] magnetometry,[3] mass detection,[4] or filtering for telecommunication devices[5] stimulate continuous efforts to improve performance of the oscillators[6,7] and to upgrade the drive-detection scheme.[8,9] An optical detection technique, widely employed in force microscopy, was also used to achieve a force resolution of 5.6×10^{-18} N[10] and mass sensitivity of 10^{-12} g.[4] The displacement of the cantilever was converted into an electric signal using a laser beam in an interferometric or beam-deflection technique followed by signal processing with an electric circuit. Another approach may be useful, however, when a small periodic force must be detected. In this approach, mechanical vibrations, induced by a weak force can be amplified by the parametric mechanism[11] and the enhanced vibrations will be detected optically. Since a ''mechanical parametric preamplifier'' can be noise-free down to the quantum-mechanical level,[12] in principle, it should greatly improve the signal-to-noise ratio of the resulting signal. A mechanical oscillator embedded in a degenerate parametric amplification scheme is also fundamentally interesting because mechanical squeezed states can be produced by such a system: the thermal vibration in one phase of the response can be reduced below thermal equilibrium level.[13]

Electrostatic,[13–15] magnetic,[16] and mechanical[17] pumping for parametric amplification in MEMS oscillations have been previously reported. In this letter, we present the realization of an optical method to provide mechanical parametric amplification in MEMS. In our system, a modulated intensity focused laser beam induces periodic (at double frequency) changes of the effective spring constant of the oscillator. The dc component of the laser beam is used to detect the vibration (at the fundamental frequency) by an interferometric scheme. Optical pumping for parametric amplification does not require the oscillator to be conducting and does not need additional coils or electrodes located in proximity to the oscillator. It can be easily integrated into existing, widely used optical detection techniques.

The single-crystal silicon oscillators were fabricated as discs supported by a pillar at the center point [Figs. 1(a) and 1(b)]. Silicon-on-insulator wafers with a 250-nm-thick silicon layer on top of a 1 μm SiO_2 layer were used for the microfabrication. Disks of radius R from 5 to 20 μm were defined by electron-beam lithography followed by a dry etch through the top silicon layer. Dipping the resulting structure into hydrofluoric acid undercuts the silicon oxide starting from the disk's periphery toward the center. By timing this wet etch, the diameter of the remaining column of the silicon oxide, which supports the released silicon disk, can be varied. In this letter, we present data obtained with disks of radius 20 μm, supported by the SiO_2 pillars of diameter 6.7 μm.

After high pressure CO_2 critical point drying, the samples were mounted on a piezoceramic transducer and placed into a high vacuum chamber ($P\sim 10^{-7}$ Torr). An ac voltage applied to the piezotransducer was used to excite the MEMS oscillators. The vibration was detected by an interferometric technique with a HeNe laser beam. The light intensity was controlled by an electro-optical modulator (EOM). All the measurements were done at room temperature.

When considering the modes of vibration, our oscillator can be regarded as an annular disk plate clamped on the inner radius and free on its periphery.[18] Figure 1(c) illustrates three modes of vibrations corresponding to γ_{01}, γ_{00}, and γ_{02} in Ref. 19, and marked as 1, 2, 3, respectively. For our structure, the highest quality factor 11 000 was observed for the second mode, γ_{00}, of the disk with a frequency of 0.89 MHz. Results presented later in this letter pertain to this particular resonance.

Figure 2 shows that a significant change of the resonant frequency of the disk ($\Delta f\sim 300$ Hz) was observed when the dc power of the laser beam, focused at the circumference of the disk was increased up to 260 μW. We attribute the frequency increase to mechanical stress, induced within the oscillator by the local heating and thermal expansion. The scaling of frequency with temperature can be demonstrated by

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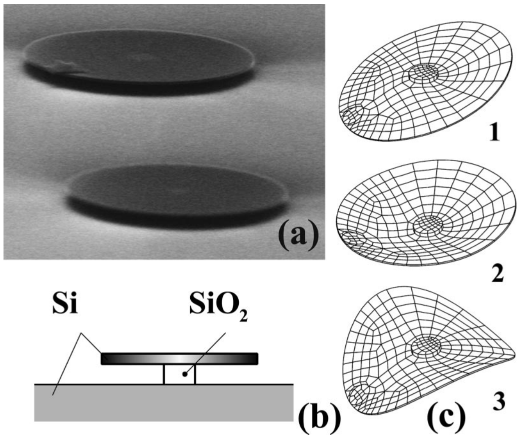

FIG. 1. Scanning electron micrograph of the silicon disk oscillators (a), cross section of the device (b). First three modes of oscillations are shown, arranged by frequency $f_3>f_2>f_1$ (c).

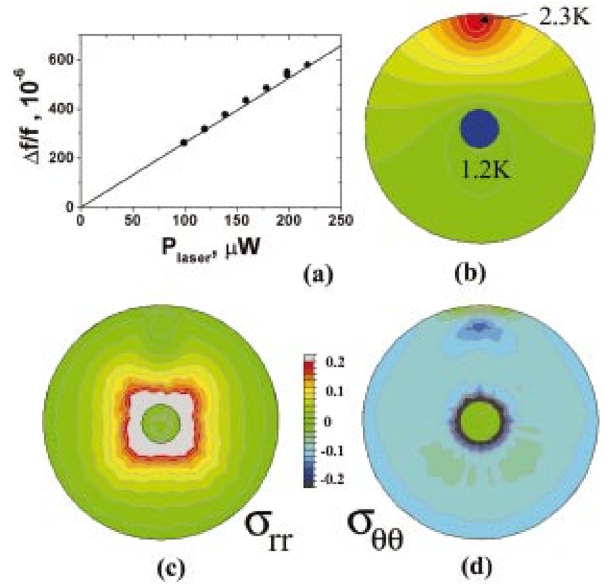

FIG. 2. (Color) Relative frequency shift versus incident dc laser power. Experimental data points (black circles) and the result of FEM calculations (solid line) are presented (a). Two-dimensional temperature distribution over the oscillator computed by FEM. The model assumes 25% absorption of 260 μW dc laser power over 5 μm diameter spot at the edge of disk. The contour spacing is 0.1 °C (b). Two-dimensional stress distribution for the radial σ_{rr} (c) and hoop $\sigma_{\theta\theta}$ (d) components computed by FEM for the same heating conditions as in Fig. 2(b). Stress is normalized by value $\alpha E \Delta T$ (α is thermal expansion coefficient, E is Young's modulus, and ΔT is the maximum temperature change across the disk due to heating). Gray and red corresponds to tensile stress while blue regions show compressive stress.

analogy with the problem of a pinned beam under thermal stress. In this case, stress caused by thermal expansion shifts the frequency of the beam according to

$$\frac{1}{f}\frac{df}{dT} = -\frac{6\alpha L^2}{\pi^2 h^2}, \tag{1}$$

where f is frequency, T is temperature, α is thermal expansion coefficient, and h and L are the beam thickness and length, respectively.[20] Note that in a disk the radial and hoop components of stress change in opposition to each other, so that unlike a beam, and depending on the mode of vibration, one can increase or decrease disk frequencies by heating.

Numerical simulations were used to analyze the frequency shift for our experimental situation in which the silicon disk is inhomogeneously heated by a focused laser beam. Figures 2(b), 2(c), and 2(d) demonstrate the two-dimensional temperature and stress distributions obtained by finite element analysis (FEM) for a 260 μW laser beam, focused to a spot size of 5 μm, assuming a light absorption coefficient of 25%.[21] The FEM solution modeled both the Si disk and the oxide base. Both the temperature variation and displacement were taken to be zero at the bottom of the oxide base. The maximum temperature rise at the center of the laser beam was estimated at $\Delta T(260\ \mu\mathrm{W}) = 2.35\,°\mathrm{C}$. The resulting stresses in the radial direction σ_{rr} were found to be primarily tensile, while in the hoop direction, $\sigma_{\theta\theta}$ is primarily compressive. The tensile radial stresses increase the frequency of modes with primarily radial bending. The experimentally observed linear dependence of the frequency as a function of dc laser power is in a good agreement with the results of the FEM [solid line in Fig. 2(a)], and with the theoretical prediction of Eq. (1).

Control of the effective spring constant, k, of the oscillator by the laser beam opens the path to parametric amplification. EOM was used to partially modulate the intensity of the laser beam, which in turn, causes a periodic change of the stiffness of the oscillator. In order to produce parametric amplification, an increase of the spring constant Δk should occur twice per period, when the deflection of the oscillator is at its maximum, contributing to potential energy $E_p = (k_0 + \Delta k)x^2/2$. In our experiment, synchronization of the stiffness modulation with the motion of the oscillator was achieved by using the ac piezodrive voltage V_{piezo} as a reference signal. An external generator provided a double frequency output V_{mod}, phase locked to the V_{piezo}. After amplification, V_{mod} was used to control the EOM, providing double-frequency modulation of the beam intensity with controlled depth of modulation and phase shift with respect to the reference signal V_{piezo}.

Figure 3(a) demonstrates the increase of the amplitude of the oscillator as a function of the amplitude of the ac component in the laser power. The phase shift ($\varphi = 90°$) was chosen to provide maximum amplification. The gain was defined as the ratio of the vibration amplitude with laser pumping on and the amplitude of the solely piezodriven vibrations (when the laser beam has a dc component only). A 30 times increase of the oscillation amplitude was observed while the modulation of the laser power was increased from zero to $P_{\mathrm{mod}} = 90\ \mu$W (the amplitude of the local temperature variation under the laser beam was estimated by FEM as 0.25 °C). When the pumping power was increased further, self-generation was observed.

The thin line in Fig. 3(a) shows the result of calculations based on the expression for the amplitude of the parametrically amplified oscillations[13]

$$A = F_0 \frac{Q\omega_0}{k_0}\left(\frac{\cos\varphi}{1+Q\Delta k/2k_0} + j\frac{\sin\varphi}{1-Q\Delta k/2k_0}\right). \tag{2}$$

Here F_0 is the amplitude of the driving force, $F(t) = F_0\cos(\omega_0 t + \varphi)$, Q is quality factor of the oscillator, and the time-varying spring constant is $k(t) = k_0 + \Delta k \sin(2\omega_0 t)$. From the results presented in Fig. 2(a) we assume a linear dependence of the spring modulation Δk as a function of the

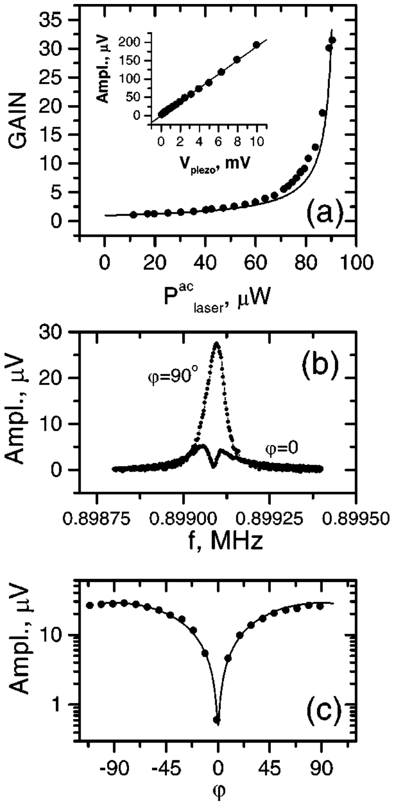

FIG. 3. The parametric amplification gain versus amplitude of the laser power modulation. Experimental points are shown by black circles. The solid line is a fit according to Eq. (2). The insert shows that the amplified vibrations behave linearly with the piezodriving voltage at a constant optical pump (a). Resonance curves obtained for orthogonal phases of the optical pumping ($\phi=0$, $\phi=90°$) (b). Amplitude of the resonance as a function of the phase shift between piezodrive voltage and optical pump is shown together with theoretical fit (solid line) (c).

laser power modulation $P_{\text{laser}}^{\text{ac}}$. The slope $d(\Delta k)/dP_{\text{laser}}^{\text{ac}}$ was used as a fitting parameter. The insert in Fig. 3(a) demonstrates that for the fixed pumping the amplitude of the oscillations is a linear function of the piezodrive voltage, which means that gain is independent of the level of vibration.

The effect of the phase shift, φ, between the piezodrive and the optical pump is illustrated in Figs. 3(b) and 3(c). The resonant response for $\varphi=90°$ (maximum amplification) and $\varphi=0$ are shown in Fig. 3(b). The suppression of the vibrations at a phase $\varphi=0$ is clearly seen. The amplitude of oscillation as a function of the phase shift φ, shown in Fig. 3(c) was fitted by the theoretical expression that follows from Eq. (2). Excellent agreement between experimental data (black circles) and the theoretical prediction (solid line) was found.

The enhanced sensitivity of degenerate parametric amplifier could find many applications, providing the reference signal can be used to define the phase shift. For example, in magnetic resonance force microscopy (MRFM)[22] modulation of the rf magnetic field provides such an ''internal clock.'' For ultrasensitive experiments like MRFM, the requirement of a high vacuum that is desirable for optical pumping does not impose additional restrictions. Also, according to recent reports,[7] the quality factor of the silicon micromechanical oscillators can be improved by an order of magnitude by a high temperature treatment in UHV. This provides a wide area of applications for optical detection of resonant MEMS enhanced by parametric amplification.

In conclusion, high frequency ($f\sim1$ MHz), high quality factor ($Q=11\,000$) single crystal silicon micromechanical oscillators were fabricated with a cylindrical symmetry design. It was shown that a low power ($P\sim100$ μW) laser beam focused at the periphery of the disk can tune the resonant frequency of the oscillator. Using double-frequency modulation of the laser beam power, we have achieved parametric amplification of the mechanical vibrations. A gain of up to 30 and the expected phase dependence of a parametric amplifier were demonstrated. We believe this method can greatly enhance the sensitivity of MEMS used for small force detection.

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