Parametric amplification in a torsional microresonator

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We observe parametric amplification in a torsional micron-scale mechanical resonator. An applied voltage is used to make a dynamic change to the torsional spring constant. Oscillating the spring constant at twice the resonant frequency results in a phase dependent amplification of the resonant motion. Our results agree well with the theory of parametric amplification. By taking swept frequency measurements, we observe interesting structure in the resonant response curves. © 2000 American Institute of Physics. 

Micromechanical torsional oscillators are highly useful tools in the physical sciences. The rotational motion makes these structures ideal for studies of magnetism and vortex physics. They have also been used to detect very small charges. The detection capabilities can be enhanced by taking advantage of dynamic behaviors that have not previously been observed in torsional oscillating systems. We describe a simple system in which the torsional spring constant of a micron-scale oscillator can be directly modified with an applied voltage. A modulation of the spring constant at twice the resonant frequency amplifies the resonant motion of the structure. This type of parametric amplification has been previously observed in cantilever oscillators. These measurements were performed on extremely small resonators, with torsion members that have widths below 200 nm. The results shown, however, are applicable to structures of any size scale. Our measurements reveal interesting properties about the frequency response of parametrically amplified systems.

The expected behaviors of a mechanical parametric amplifier have been derived previously. The predictions of this theory are presented here for later comparison to the actual data. Consider a system that is described by the following equation of motion:

\[ I \ddot{\varphi} = -[\kappa - \kappa'(t)] \varphi - \frac{I \omega_0}{Q} \varphi + \tau(\omega t), \]

where \( \varphi \) is angular displacement of the oscillator from equilibrium, \( \kappa \) is the time independent (normal) part of the torsional spring constant, and \( \kappa'(t) \) is a dynamic modulation of the spring constant that is brought about by the application of a time dependent potential at exactly twice the driving frequency, \( 2\omega_0 \). \( I \) is the moment of inertia, \( Q \) is the quality factor, \( \omega_0 \) is the resonant frequency of the oscillator, and \( \tau(\omega t) \) is the applied torque at frequency \( \omega \). If we drive the structure on resonance with an applied torque given by

\[ \tau(t) = \tau_0 \sin(\omega t + \theta), \]

with \( \theta \) the phase between the \( \omega \) and \( 2\omega \) components with the result is a modulation (or time dependence) of the spring constant given by

\[ \kappa'(t) = \kappa_0 \cos(2\omega_0 t). \]

Theory predicts that the system will respond with an amplitude of oscillation, \( \varphi_0 \), described by

\[ \varphi_0 = \frac{\tau_0 Q}{\kappa} \left[ \frac{\cos^2 \theta}{(1 + Q \kappa_0^2/2\kappa)^2} + \frac{\sin^2 \theta}{(1 - Q \kappa_0^2/2\kappa)^2} \right]^{1/2}. \]

The first factor is the right hand side, \( (\tau_0 Q)/(\kappa) \), is just the normal resonant response of an oscillator when \( \kappa'(t) = 0 \), i.e., when no \( 2\omega \) signal is applied. The second factor then acts as a phase-dependent gain. When the phase \( \theta = 0 \), the system will be deamplified. When the phase \( \theta = \pi/2 \), the gain diverges to infinity when \( \kappa_0^2/(2\kappa) \).

It is possible to fabricate a torsional microresonator that is well described by the earlier equations of motion. We have previously reported on the fabrication and measurement techniques used in the study of very small silicon oscillators. Commerically available silicon on insulator wafers are patterned using electron beam lithography. The pattern is transferred into the top silicon surface using a reactive ion etch, and the small features are released in an isotropic wet etch that selectively attacks the buried oxide. We have studied the behavior of structures such as the one shown in Fig. 1. This consists of a rectangular paddle symmetrically suspended by narrow beams that are typically 150–200 nm in width. The length and width were varied over a large range. The minimum size of the paddles in which we observed parametric amplification was \( 2 \times 2 \mu \text{m}^2 \) and the largest was \( 4 \times 25 \mu \text{m}^2 \). All of the data in this letter is from a \( 4 \times 15 \mu \text{m}^2 \) paddle. Thin layers of Cr (5 nm) and Au (10 nm) are evaporated on the top surface and the underlying substrate so that electrical connection can be made. The structures to be tested are held under vacuum to avoid viscous damping effects. Motion is detected optically. Light from a He–Ne laser (632.8 nm) passes through a quartz window into the vacuum chamber and is focused onto the sur-

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face of the oscillator. Motion of the structure will result in a
change in the reflectance due to interference, and this change is
detected using an alternating current (ac)-coupled photo-
receiver. The structures are driven electrostatically by applying
a potential between the top and bottom surfaces. The substrate
is grounded while the suspended structure is driven
with an ac signal (with \( \omega \) and \( 2\omega \) components) that is added
to a direct current (dc) voltage.

In earlier work, we have shown that the torsional mode
can be excited in this fashion, even though the total torque
on such a symmetric oscillator should be zero.\(^{10}\) Slight asym-
metries are incurred during the fabrication process and these
are sufficient to allow the excitation of torsional motion. We
have also shown that a small applied dc voltage will produce
a substantial decrease in the effective spring constant, a prop-
erty that enables parametric amplification of the motion.
From these results, we know that if the structure is placed
under a dc voltage, \( V_{dc} \), which is added to an ac signal,
\( V_{ac}\sin 2\omega t \), then the spring constant will be pumped by an
amount

\[
\kappa(t) = \frac{\varepsilon_0 w^3 l^3 V_{dc} V_{ac}}{12d^4} \cos(2\omega t),
\]

where \( \varepsilon_0 \) is the permittivity of free space, and \( l, w, \) and \( d \)
are shown in Fig. 1. If we let \( V' = (24\kappa d^3)/(Q\varepsilon_0 w^3 l^3 V_{dc}) \)
and \( A_0 = (\tau Q)/(\kappa) \), then using Eqs. (3), (4), and (5) we show
that the resonant response as a function of the ac signal
amplitude is

\[
\varphi_0 = A_0 \left[ \frac{\cos^2 \theta}{(1 + V_{ac}/V')^2} + \frac{\sin^2 \theta}{(1 - V_{ac}/V')^2} \right]^{1/2}.
\]

To observe this effect, we drive the structures by applying
a torque at a frequency \( \omega \) and oscillate the spring con-
stant at \( 2\omega \). The application of a voltage at \( 2\omega \) will also
produce a torque at that frequency. For an oscillator with a
sufficiently high \( Q \), this torque will not affect the response of the
oscillator at \( \omega \). The optical response at the driving fre-
quency is measured using a spectrum analyzer. The fre-
quency, \( \omega \), is swept across a range that allows us to observe
the resonant peak shape of the response.

Figure 2 shows the parametric amplification effect. A
series of curves are shown in which the signal at the driving
frequency is constant while the double frequency pumping
signal varies over the range shown in the legend. The peak
amplitudes as a function of the pumping signal are plotted
and these values are fitted to a function of the form of Eq.
(6). From the fit, we get a value of 0.20 +/- 0.01 V for the
asymptotic voltage \( V' \). The predicted value is 0.20 +/- 0.05 V.

These swept frequency measurements also show some
behavior characteristics of parametric amplification that are
not generally discussed. As the amplification increases, the
peak width becomes substantially narrower and the peak as-
sumes a form that is non-Lorentzian. This is due to the phase
dependence of the amplification. The response to frequencies
on either side of the resonance reflects the fact that the an-
gular excursion and the pump (at \( 2\omega \)) are of a different phase
from that described in Eq. (4) and thus the amplitude at these
frequencies will be deamplified. This is well illustrated in
Fig. 3 in which several resonance curves are normalized to their
respective peak values. The peak with the highest am-
plification has the narrowest width.
Another interesting feature is shown in Fig. 4. We have varied the phase of the pump relative to the drive while the pump and drive amplitudes are both held constant. An arbitrary phase offset angle \( \sim 120^\circ \) is incurred in the signal generation. As the phase is varied by 90°, we see the response evolve from the amplified state to the deamplified state. When the system is under deamplification, a dip in the response is observed at the resonant frequency. This is, again, a result of the phase dependence. The response is deamplified at the resonant frequency, but the sidebands are now amplified.

This amplification technique should allow a mechanical resonator to achieve a narrower bandwidth than it would be capable of in the unpumped state. The phase dependence of the amplification will lead to an increased sensitivity to phase noise. Our purpose in this letter is to demonstrate that parametric amplification can be realized in torsional systems. Because of their simplicity, these parametric amplification techniques could be extended to coupled mechanical oscillators whose coupling spring constant can be dynamically altered.\(^1\) Electronic amplifiers of this type are well known for their low noise figure.

Parametric techniques will be critical in high frequency applications where the performance of mechanical devices become limited by processes such as thermoelastic and phonon–phonon effects that intrinsically limit the \( \omega Q \) product.\(^2,3\) We have demonstrated that torsional devices are capable of producing a response that is nearly an order of magnitude narrower than that achieved without parametric amplification.

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\(^3\)E. Thompson, G. Lawes, J. M. Parpia, and R. O. Pohl (unpublished).
\(^12\)See for example, V. B. Braginsky, V. P. Mitranov, and V. I. Panov, \textit{Systems with Small Dissipation} (University of Chicago Press, Chicago, 1985), Chap. 2.