In superfluid hydrodynamics, the presence of two interpenetrating “fluids,” the normal and superfluid components, leads to a low velocity mode—second sound—in which the two components move independently [1]. Apart from being a manifestation of the special properties of superfluidity, it provides a valuable measurement tool with which the density of the superfluid component, ρs, can be directly determined [2].

Liquid 3He in highly porous aerogel glass is the first realization of disordered superfluid 3He [3,4]. In contrast to superfluid 4He in aerogel [5], its zero-temperature superfluid fraction, ρs/ρ, is substantially suppressed from unity. Moreover, by changing the pressure [6] or the microscopic structure of the aerogel [7] one can suppress ρs/ρ and Tc to zero. Direct measurements of the superfluid fraction with a torsional pendulum exhibited interference with sound modes (including those reported in this Letter) of 3He in aerogel [3,7,8]. We describe sound modes that we observed and the use of acoustic spectroscopy as an alternative technique for the accurate determination of ρs/ρ.

Our resonator consisted of a cylindrical aerogel sample (length L = 1.59 cm, radius R = 0.48 cm) grown inside a stainless steel tube. Brass diaphragms of 0.3 mm thickness (to which piezoceramic speaker and microphone transducers were attached) were pressed against the aerogel. At room temperature the gap between the aerogel sample and the tube was small, but the aerogel could freely slide within the tube. Sound spectra were recorded as the quadrature response of the microphone to the oscillations of the speaker while the drive frequency was swept continuously. The temperature, T, was determined with a melting curve thermometer.

The aerogel sample is a conglomerate of ~50 Å silica particles that form a disordered network of strands with a distribution of interstrand spacing between ~50 Å and ~1000 Å, porosity φ = 0.98, density ρs = ρSiO2(1 − φ) = 0.04 g/cm³, and with a longitudinal sound velocity c₀ ~ 50 m/s. In liquid 3He, the viscous penetration depth, δv, at a frequency, f, is

\[ \delta_v = \left( \frac{\eta}{\pi f \rho_s} \right)^{1/2}, \]  

where η is the viscosity of 3He, ρn is the density of the normal component, and ρs + ρn = ρ = φρbulk is the density of 3He in aerogel. At the bulk superfluid transition temperature, Tcbulk, and frequencies below 10 kHz, δv ≃ 0.1 mm. Hence, in aerogel the normal component is always viscously clamped to the silica skeleton. If the latter were rigid, only the superfluid component would move and allow a pressure wave, “fourth sound,” with velocity, c₄,

\[ c_4^2 = \frac{\rho s}{\rho} c_1^2 + \frac{\rho n}{\rho} c_2^2, \]  

to propagate. Here \( c_1 = \frac{(\partial \rho / \partial T)_T}{(\partial \rho / \partial T)_T} \) is the velocity of the ordinary compression wave (first sound), and \( c_2 = S(T \rho_s / C_p \rho_n)^{1/2} \) is the second sound velocity which, in 3He-B, is less than 3 cm/s [9].

Because the skeleton is elastic, the normal component can oscillate with the silica and will exhibit a renormalized mass and stiffness due to the aerogel’s presence. This system is directly connected to the concept of a “super-solid” in which a persistent mass current (for example, the coherent flow of mobile vacancies in a quantum crystal [10]) can be established relative to the lattice. As a consequence of conservation of the momentum of the solid and the mass current, four collective modes (sounds) are predicted for this system, resembling one longitudinal and two transverse elastic waves in the solid, together with a superfluid density wave [10,11].

McKenna et al. [12] modified the conventional two fluid hydrodynamic equations to take into account coupling of the normal component to the aerogel mass and elasticity. Their equations contain a vector restoring force and no explicit shear stress tensor. They are therefore exact only for longitudinal waves in infinite media and do not allow transverse wave solutions. The velocities of both
longitudinal sounds (fast, $c_f$, and slow, $c_s$), $c_x = (c_f, c_s)$, satisfy the equation [12]

$$ (c_x^2 - c_1^2) (c_x^2 - c_2^2) + \frac{\rho_a}{\rho_n} (c_x^2 - c_a^2) (c_x^2 - c_0^2) = 0. \tag{3} $$

They also observed the propagation of both the fast and the slow modes in a $^4$He filled cylinder of aerogel. Using the bulk values for $\rho_n(T)$ (which are close to those in aerogel [5]) they found good agreement of the model with the observed temperature dependence of the sound modes.

The fast mode is a compression wave in helium-filled aerogel. In contrast, the slow mode is the analog of second sound in bulk helium and fourth sound in porous media— it exists only in the superfluid state and its velocity vanishes at $T_c$. This mode can be visualized as a small oscillation of the aerogel combined with a simultaneous out-of-phase motion of the superfluid component. The analysis of this slow mode in terms of the superfluid component also oscillates which the superfluid and normal components also oscillate out of phase, in the slow mode the restoring force exerted on the normal component is mainly determined by the elastic energy of the aerogel and not the thermal energy of the $^3$He. The scale of $c_s$ is set by the longitudinal sound velocity in aerogel, $c_a \gg c_2$ [14] [Eq. (5)]. This is also different from fourth sound in a rigid porous medium (a pure pressure wave), where the scale is set by the sound velocity in liquid $^3$He, $c_1 \gg c_a$ [Eq. (2)]. In all previous studies of $^3$He in aerogel, the aerogel played the role of quenched disorder that suppresses the amount of the superfluid component. In the present study, we observe that the aerogel also changes the dynamic properties of the normal component, giving rise to an unusual effect—the coexistence of two sound modes, each of which is a pressure wave.

With our apparatus we observed fundamental resonances of the fast and slow modes at frequencies, $f_0$, in the range 10 Hz–10 kHz. Providing they correspond to a standing wave of wavelength $\lambda = 2L$, we define the effective sound velocity, $c_s' = 2Lf_0$. The latter is related to the sound velocity in an infinite medium, $c_s$, by $c_s' = A_x c_s$, where the numerical factor, $A_x = (A_f, A_s, A_y)$, depends on the mode (fast, slow, or in empty aerogel, respectively) and boundary conditions [16]. For a pure compression wave (as would be the case of liquid in a tube), $c_s' = c_s$, and hence $A_x = 1$.

The evolution with temperature of the fundamental resonance of the fast mode is shown in Fig. 1. The sound velocity, $c_f'$, is 80% of the bulk $^3$He first sound velocity. In the superfluid phase, $c_f'$ changes by only $\sim 1\%$ or less. When all $^3$He in aerogel is normal, the solution of Eq. (3) is given by

$$ c_f^2 = \left( \frac{c_s^2 + \frac{\rho_a}{\rho} c_a^2}{1 + \frac{\rho_a}{\rho}} \right)^{-1}. \tag{4} $$

The aerogel contributes little to the restoring force, which is dominated by the compressibility of the helium, and only adds its mass, producing a decrease in the sound speed. The difference between $c_f'$ and $c_f$ calculated using Eq. (6) was less than 1% at $p = 5$ bar and 7% at 29 bar, using $^3$He properties tabulated by Wheatley [17]. The agreement is improved if we account for the change in the sound path, $L$, caused by the deformation of the diaphragms with pressure [18]. We find $A_f = 1.01 \pm 0.01$.

Examples of the slow mode resonances at different temperatures are shown in Fig. 2. To determine the center-of-peak frequency, $f_0$, they were fitted to a Lorentzian. In Fig. 3 we plot the sound velocity, $c_s'$, using data similar to that of Fig. 2 for pressures between 5 and 29 bar. The transition temperatures, $T_c(p)$, indicated with arrows in Fig. 3, are in good agreement with those from previous studies of $^3$He in this type of aerogel [4,6,7].

We note that at a temperature higher than $T_c$ (but below $T_{c}^{bulk}$) the apparent velocity of the slow mode does not vanish completely even though all the $^3$He in aerogel is in the normal state [see Figs. 2(b) and 3]. A similar
FIG. 2. (a) Spectra of the quadrature response of the slow mode ($p = 21.7$ bar) offset vertically with temperature. The sharp positive peak ($f_0$) disappearing at $T_c = 1.85$ mK is the fundamental slow mode resonance. The stronger and broader negative peak vanishing at $T_{\text{bulk}} = 2.28$ mK is the Helmholtz resonance. Five higher modes with the same $T_c$ are seen in the figure (two cross the Helmholtz mode at 1.2 and 1.4 mK and three rapidly changing modes cross just below $T_c$). (b) An enlarged plot of the low frequency region [at 10 times the drive used in (a)] shows the conversion of the slow mode into the slowly varying edge mode that disappears at $T_{\text{bulk}}$.

effect was reported by Mulders et al. [19] for superfluid $^4$He in aerogel, and was attributed to $^4$He in connected macroscopic voids in the aerogel. As our sample is believed to be free of voids, it is most likely that we excite an oscillatory “edge mode.” In such a mode, compression and lateral deformation of the aerogel stimulate longitudinal oscillations of the bulk superfluid $^3$He in the narrow gap between the aerogel and the walls of the surrounding tube. In this oscillatory motion, the inertial term is dominated by the superfluid density in the gap, and the resonant frequency should vary as the square root of bulk $\rho_s/\rho$. The expected temperature dependence agrees well with the observed frequency dependence and is shown as the solid line in Fig. 4.

In Fig. 5 we plot the superfluid fraction of $^3$He in aerogel vs reduced temperature $T/T_c$. Because of its quadratic dependence on $c_s^2$ [Eq. (5)], the edge mode contribution to $\rho_s/\rho$ is less than 0.003 at all pressures and the uncertainty of $\rho_s/\rho$ introduced by its presence is small. The values of $\rho_s/\rho$ are comparable to those obtained with a torsional oscillator [3,6–8]. The superfluid fraction extrapolates to $\rho_s/\rho(T = 0) = 0.3$ at $p = 29$ bar and to lower values at reduced pressures.

Figure 2(a) shows a Helmholtz resonance at frequency $f_H$ associated with the volume of the cell and the filling tube of length $l = 4$ cm and diameter $d = 0.14$ cm. The ratios $f_H/c_1$ vs $T/T_{\text{bulk}}$ fall on a pressure independent curve. Below 100 Hz, $f_H(T)$ closely follows the behavior expected for fourth sound, Eq. (2). At low frequencies, $\delta_v$ [Eq. (1)] is comparable to the diameter of the fill tube,
mapping of slow mode frequency provides a cleaner technique for the disordered superfluid and an elastic solid. Tracking the system that simultaneously displays behavior of a temperature. The two modes establish the unique character the other which shows a frequency shift below this temperature. The complete data set of $\rho_s/\rho$ plotted vs the reduced temperature $T/T_c$.

\[ \delta_v \sim d, \text{ so that only the superfluid component can oscillate. Hence the frequency, } f_H, \text{ is given by } [21] \]

\[ 2\pi f_H = c_4(d/D)(IL)^{-1/2} \approx c_1(\rho_s^{\text{bulk}}/\rho_{\text{bulk}})^{1/2}. \quad (7) \]

In summary, we have observed two distinct acoustic modes of superfluid $^3$He in aerogel, one which exists only below the suppressed transition temperature of $^3$He, and the other which shows a frequency shift below this temperature. The two modes establish the unique character of this system that simultaneously displays behavior of a disordered superfluid and an elastic solid. Tracking the slow mode frequency provides a cleaner technique for the mapping of $\rho_s/\rho$ than the torsional oscillator because interfering resonances are excluded. We have also observed a new edge mode which occurs at the interface between a normal fluid and a superfluid. Absolute measurements of $\rho_s/\rho$ with this technique will require computation of the exact vibrational modes of a resonator together with predetermined boundary conditions.

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[14] In aerogel, ~2% of $^3$He atoms are localized on silica strands and possess considerable spin entropy [15]. Evaluation of $c_3 = S(T_p/C_p)\rho_2^2$ for $T = 1 \text{ mK}, p = 29 \text{ bar}$, and $\rho_s/\rho = 0.3$, and accounting for $S$ and $C_p$ from localized spins, yields $c_3 \approx 0.2 \text{ m/s}$ which is higher than the bulk value [9], but still much less than $c_a \sim 50 \text{ m/s}$.
[16] The relation of $c_s^* \text{ to } c_s \text{ requires the solution of the hydrodynamic equations} [12] \text{ modified to account for the shear stress tensor with appropriate boundary conditions. However, for thin rods } (R \ll \lambda), \text{ the fundamental mode should be similar to a longitudinal wave and } c_s^* = \lambda c_s. \text{ It is important that for a particular boundary condition, } A_s = A_c. \text{ Thus, because } \rho_s/\rho \text{ depends only on the ratio } c_s/c_a \text{ [Eq. (5)], the particular value of } A_s \text{ is unimportant and the extraction of } \rho_s/\rho \text{ from } c_s^*/c_a^* \text{ is tractable. For our aerogel we found } c_s^* = 51 \text{ m/s from the in situ measured } f_0 = 1620 \text{ Hz for the empty aerogel at } T = 5 \text{ K.}
[18] J. Beamish (private communication).