Knudsen Flow and Effective Shear Viscosity of Liquid ^{3}He

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We present new torsional oscillator data on the shear viscosity of liquid ^{3}He at low temperatures. They are analyzed with a theory of Poiseuille flow, which is also applicable in regimes of temperature and pressure where the Knudsen number, characterizing the flow of the thermal excitations, may become large.

1. INTRODUCTION.

The shear viscosity η of an infinitely extended isotropic Fermi superfluid at low temperatures can be understood as arising from two body collisions in the dilute gas of thermal excitations, the Bogoliubov quasiparticles (BQP) which are specified by the (normal fluid) density ρ_n, the rms group velocity $\bar{v}$ and the (viscous) mean free path λ_η and is given by the simple gas kinetic expression [1]:

$$\eta = \frac{1}{5}\,\rho_n\,\bar{v}\,\lambda_\eta \tag{1}$$

Any experiment attempting to measure η introduces, however, the sample container walls (for example the parallel plates, a distance d apart, of a torsional oscillator), from which the BQP may be scattered off in a way that depends on the surface structure (diffuse, specular, etc. scattering) and, in a system with non-s-wave Cooper pairing like superfluid ^{3}He, on the amount of gap suppression near the walls leading to a quantum mechanical "retroflection" (characterized by momentum conservation and a complete reversal of the group velocity) of quasiparticles (Andreev reflection [2] or particle hole conversion [3]). The Knudsen number Kn = λ_η/d measures the relative importance of two particle collisions with respect to wall scattering effects. In ref. [4] we have demonstrated that the high pressure ^{3}He-B viscosity data of Archie et al. [5] can be interpreted as arising from both diffuse and Andreev boundary scattering of the quasiparticle system close to the Knudsen transition. It is the purpose of this paper to present and analyze new experimental shear viscosity data of superfluid ^{3}He-B, taken in regimes of both small (high pressure, temperature) and large (low pressure, temperature) Knudsen numbers.

2. EXPERIMENT.

The experiments were carried out using torsional oscillators having a disk shaped ^{3}He-space with characteristic heights of 45 μm and 135 μm, respectively. The oscillators were operated at resonant frequencies of about 3525 and 385 Hz, respectively. The effective viscosity and normal fluid density were obtained by simultaneous measurement of the period and dissipation. The oscillators, demagnetization stage, cryostat as well as the hydrodynamics have been described elsewhere [6].

3. THEORY.

Our theoretical description starts from a microscopic Landau-Boltzmann equation for BQP derived in ref. [1] using a well - controlled relaxation time approximation for the collision integral, consistent with (i) conservation of quasiparticle momentum and (ii) relaxation of the stress tensor on the proper (viscous relaxation) time scale $\lambda_\eta/\bar{v}$. The pressure variation of the resulting macroscopic quantities originates from that of the transition temperature, the zero temperature gap as well as of a few Landau and scattering parameters. The presence of walls is accounted for by a general boundary condition, i.e a linear relationship between incoming and reflected BQP partial currents:

$$|v_{kz}|\delta f_k^{>} = 2\sum_{\substack{k' \\ v_{k'z}<0}} W_{k,k'}|v_{k'z}|\delta f_{k'}^{<} \tag{2}$$

Here $W_{k,k'}$ represents a general probability amplitude for elastic wall scattering which is assumed to ensure quasiparticle number conservation and to guarantee microscopic reversibility of the scattering process. v_{kz} is the surface normal component of the BQP group velocity. Finally δf is the linearized momentum distribution of incoming (<) and outgoing (>) BQP at the surface. As special cases for $W_{k,k'}$ we consider (i) diffuse scattering

$$W_{k,k'} = \text{azimuthally isotropic} \tag{3a}$$

and (ii) Andreev scattering of BQP from a step-like gap profile:

$$W_{k,k'} = R(E)\,\delta_{k,k'}\,\delta_{v_{kz},-v_{k'z}} \tag{3b}$$

with $R(E) = (E-\sqrt{E^2-\Delta^2})/(E+\sqrt{E^2-\Delta^2})$. Next we derive an equation for the averaged normal velocity field from the Landau Boltzmann equation which is solved with variational methods. The result is a generalized form of Hagen Poiseuille's law in which the bulk viscosity η has turned into an effective viscosity η_{eff} still of the form (1) but with the bulk viscous mean free path λ_η replaced by a length λ_η^{eff}, which fully incorporates finite size effects in that it describes the transition from particle collision dominated hydrodynamic ($\lambda_\eta^{eff}=\lambda_\eta$) to a slip ($\lambda_\eta^{eff}=\lambda_\eta(1+c_s Kn)^{-1}$) and eventually to a wall collision dominated Knudsen ($\lambda_\eta^{eff} \approx d$) regime:

$$\lambda_\eta^{eff} = \lambda_\eta\,\frac{d}{d+c(Kn;W;T)\lambda_\eta} \tag{4}$$

The first order mean free path correction to hydrodynamic behavior has been termed a slip correction because it may be associated with the macroscopic velocity field extrapolating to zero only a distance $\zeta = c_s\,\lambda_\eta$ (the slip length) behind the walls. Here c_s is a slip coefficient which only depends on the wall scattering probability W, but not on the Knudsen number. Deviations from simple slip behavior manifest themselves through a Knudsen number dependence of the quantity c which, in general, is obtained from a variational solution of the integral equation for the averaged velocity field:

$$c = \frac{12}{5}\,\overline{\mathrm{Kn}}\left(\frac{\left[\frac{1}{d}\int_{-d/2}^{d/2} dz\,\Phi(z)\right]^2}{\frac{1}{d}\int_{-d/2}^{d/2} dz\,dz'\,\Phi(z)H(z,z';W)\Phi(z')} - 1\right) - \frac{1}{\overline{\mathrm{Kn}}} \qquad (5)$$

Here $\overline{\mathrm{Kn}} = \lambda/d$ with λ the quasiparticle mean free path. We used a parabolic trial function Φ, known to be exact in both the hydrodynamic and the Knudsen limit, respectively. H is the kernel of the integral equation, the explicit form of which can be found in ref. [4].

3. DISCUSSION OF RESULTS.

In Fig. 1 we have plotted the reduced experimental viscosity data from the 135 μm oscillator for 0 and 29 bar, respectively, as full circles and from the 45 μm oscillator for 0 bar as open circles vs. inverse reduced temperature. The high pressure data are generally characterized by a sharp initial decrease below T_c followed by a tendency to level off at intermediate temperatures and eventually by a further decrease by a total of about 3 orders of magnitude until the lowest temperature is reached. In contrast, the 0 bar data show a less pronounced initial decrease and no levelling-off tendency at all resulting in a crossover with the 29 bar data. Our theoretical analysis attempts to show physical "bounds" on the effective viscosity η_{eff} which is calculated from eqs. (4) and (5). First, if backscattering effects can be ignored, the most efficient transfer of transverse BQP momentum is obtained with diffusely scattering surfaces, the results being represented by the upper of the three pairs of full (135μm) and dashed (45μm) lines, respectively. Specularly reflected BQP do not contribute to the transfer of transverse momentum resulting in an effective viscosity lower than for purely diffuse scattering. We have, however, not considered this possibility further, since normal state results seem to be consistent with the presence of diffuse scattering at high and even "super-diffuse" scattering at low pressure [7]. Second, we superimpose Andreev reflection processes to the diffuse ones according to eqs. (3a and b). The reflection amplitude R(E) is much larger in this case than for realistic gap suppression profiles and we clearly overestimate the resulting depression of the effective viscosity, shown as the lower of the three pairs of full and dashed lines, respectively. As could be anticipated from the results of ref. [4] the data are bounded by the theoretical extremes considered at high pressure. In addition the general shape of experimental and theoretical curves agrees well at all pressures and cell heights. At low pressure and for both cell heights, however, the data points lie even below the lower theoretical curves. At present, we are not able to explain this discrepancy within our theoretical model. Clearly the treatment of surface scattering will have to be reconsidered and improved. A recent selfconsistent treatment of this problem within the quasiclassical approach in which the surface roughness enters as the only adjustable parameter [8], seems to model the data better, however, only if a temperature (and pressure) dependence of the surface roughness (of presently unknown origin) is put in. The data seem indeed to imply a temperature variation of the effectiveness of diffuse wall scattering which is beyond the expectations from present theories. Aiming to understand the origin of this discrepancy, experiments with different surfaces of well defined roughness would clearly be of great help.

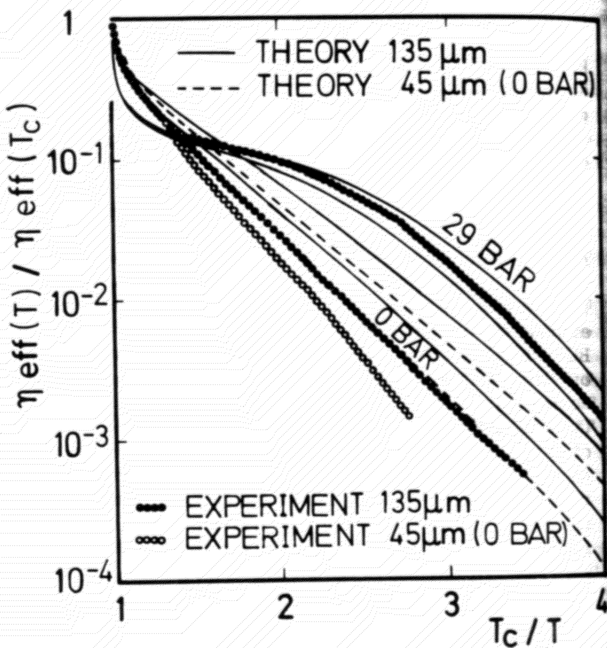

Fig. 1: Effective viscosity of superfluid ^{3}He-B at two different pressures and oscillator heights as a function of inverse reduced temperature. Symbols are explained in the text.

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